

Mathematics Revision 2006
Scope and Sequence by Grade
Kindergarten

Strand I – Patterns, Relationships, Functions

Michigan Content Standard:

1. Patterns:

- Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.

Grade Level Benchmark:

EL.1. Recognize, describe and extend numerical and geometric patterns.

Grade Level Content Expectation:

- N.ME.00.05 Count orally to 100 by ones. Count to 30 by 2's, 5's and 10's using grouped objects as needed.
- N.MR.00.10 Create, describe, and extend simple number patterns.
- G.GS.00.03 Create, describe, and extend simple geometric patterns.

EL.2. Represent and record patterns and relationships in a variety of ways including tables, charts and pictures.

EL.3. Use patterns to describe real-world phenomena.

EL.4. Explore various types of numeric and geometric patterns (repeating, growing, shrinking).

EL.5. Apply their experiences with patterns to help solve problems and explore new content.

Grade Level Content Expectation:

- G.GS.00.03 Create, describe, and extend simple geometric patterns.

Strand II: Geometry and Measurement

Michigan Content Standard:

1. Shape and Shape Relationship

- Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

Grade Level Benchmark:

EL.1. Recognize and name familiar shapes in one, two and three dimensions such as lines, rectangles and spheres and informally discuss the shape of a graph.

Grade Level Content Expectation:

- G.GS.00.01 Relate familiar three-dimensional objects inside and outside the classroom to their geometric name, e.g., ball/sphere, box/cube, soup can/cylinder, ice cream cone/cone, refrigerator/prism.

EL.2. Describe the attributes of familiar shapes.

Grade Level Content Expectation:

- G.GS.00.02 Identify, sort and classify objects by attribute and identify objects that do not belong in a particular group.

EL.3. Compare, sort and classify familiar shapes.

Grade Level Content Expectation:

- G.GS.00.01 Relate familiar three-dimensional objects inside and outside the classroom to their geometric name, e.g., ball/sphere, box/cube, soup can/cylinder, ice cream cone/cone, refrigerator/prism.
- G.GS.00.02 Identify, sort and classify objects by attribute and identify objects that do not belong in a particular group.

EL.4. Drawing and constructing shapes in two and three dimensions are important ways to represent the world.

Grade Level Content Expectation:

- G.GS.00.03 Create, describe, and extend simple geometric patterns.

EL.5. Explore ways to combine, dissect, and transform shapes.

EL.7. Use shape, shape properties, and shape relationships to describe the physical world and to solve problems.

Grade Level Content Expectation:

- G.GS.00.01 Relate familiar three-dimensional objects inside and outside the classroom to their geometric name, e.g., ball/sphere, box/cube, soup can/cylinder, ice cream cone/cone, refrigerator/prism.

Michigan Curriculum Standard

2. Position

- Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

Grade Level Benchmark:

EL.1. Locate and describe objects in terms of their position, including front, back, inside, outside, right, left, over, under, next to, between and locations on the number line on a coordinate graph and on a map.

EL.2. Locate and describe objects in terms of their orientation, direction and relative position, including up, down, back, N-S-E-W, flipped, turned, translated; recognize symmetrical objects and identify their lines of symmetry.

EL.5. Use concepts of position, direction, and orientation to describe the physical world and to solve problems.

Michigan Curriculum Standard

3. Measurement

- Students compare attributes of two objects, or of one with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

Grade Level Benchmark:

EL.1. Compare attributes of objects; develop standard units of measurement; and select and use standard tools for measurement.

Grade Level Content Expectation:

- M.UN.00.01 Know and use the common words for the parts of the day (morning, afternoon, evening, night) and relative time (yesterday, today, tomorrow, last week, next year).
- M.TE.00.02 Identify tools that measure time (clocks measure hours and minutes; calendars measure days, weeks, and months).
- M.UN.00.03 Identify daily landmark times to the nearest hour (lunchtime is 12 o'clock, bedtime is 8 o'clock).
- M.UN.00.04 Compare two or more objects by length, weight and capacity, e.g., which is shorter, longer, taller?

- M.PS.00.05 Compare length and weight of objects by comparing to reference objects, and use terms such as shorter, longer, taller, lighter, heavier.

EL.2. Identify the attribute to be measured and select the appropriate unit of measurement for length, mass, weight, area, perimeter, capacity, time, temperature, and money.

Grade Level Content Expectation:

- M.UN.00.04 Compare two or more objects by length, weight and capacity, e.g., Which is shorter, longer, taller?

EL.3. Develop strategies for estimating measures and compare the estimates to the results of the measurement; decide if an estimate is “a good estimate.”

EL.4. Explain the meaning of measurements and recognize that the number of units it takes to measure an object is related to the size of the unit.

EL.6. Apply measurement to describe the real world and to solve problems.

Strand III: Data Analysis and Statistics

Michigan Content Standard:

1. Collection, Organization, Presentation of Data

- Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.

Grade Level Benchmark:

EL.1. Collect and explore data through counting, measuring and conducting surveys, and experiments.

EL.2. Organize data using concrete objects, pictures, tallies, tables, charts, diagrams, and graphs.

EL.3. Present data using a variety of appropriate representations and explain the meaning of the data.

Michigan Content Standard:

2. Description and Interpretations

- Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.

Grade Level Benchmark:

EL.1. Read and explain data they have collected and organized themselves and progress to reading data from other sources.

EL.2. Patterns in data distributions help students to interpret the findings.

EL.3. Students learn to draw conclusions and to convince and persuade using data to justify their positions.

Grade Level Benchmark:

EL.1. Making and testing hypotheses is an essential technique for gaining new knowledge.

Strand IV: Number Sense and Numeration

Michigan Content Standard:

1. Concepts and Properties of Numbers

- Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.

Grade Level Benchmark:

EL.1. Develop an understanding of whole numbers; and read, write and count using whole numbers; investigate basic concepts of fractions and decimals.

Grade Level Content Expectation:

- N.ME.00.01 Count whole numbers and recognize how many objects are in sets to 30.
- N.ME.00.02 Use one-to-one correspondence to compare and order sets of objects to 30 using such phrases as “same number”, “more than”, or “less than”; use counting and matching.
- N.ME.00.03 Compare and order numbers to 30 using phrases such as “more than” or “less than.”
- N.ME.00.04 Read and write numerals to 30 and connect them to the quantities they represent.

EL.2. Investigate and develop an understanding of the base 10, place-value system.

Grade Level Content Expectation:

- N.ME.00.06 Understand the numbers 1 to 30 as having one, or two, or three groups of ten and some ones. Also count by tens with objects in ten-groups, to 100.

EL.3. Develop an understanding of the properties of numbers (e.g., order) and of the properties of the special numbers 0 and 1.

EL.4. Apply their understanding of number systems to model and solve problems.

Grade Level Content Expectation:

- N.MR.00.08 Describe and make drawings to represent situations/stories involving putting together and taking apart for totals up to 10; use finger and object counting.

Michigan Content Standard:

2. Representation and Uses of Numbers

- Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Grade Level Benchmark:

EL.1. Represent whole numbers, fractions, and decimals using concrete, pictorial, and symbolic representations.

EL.2. Explore and recognize different representations for the same number and explain why they are the same.

Grade Level Content Expectation:

- N.ME.00.06 Understand the numbers 1 to 30 as having one, or two, or three groups of ten and some ones. Also count by tens with objects in ten-groups, to 100.
- N.MR.00.07 Compose and decompose numbers from 2 to 10, e.g., $5 = 4 + 1 = 2 + 3$, with attention to the additive structure of numbers, e.g., 6 is 1 more than 5, 7 is one more than 6.

EL.3. Investigate ways numbers are used (e.g., counting, ordering, naming, locating, measuring).

Grade Level Content Expectation:

- N.ME.00.01 Count whole numbers and recognize how many objects are in sets to 30.
- N.ME.00.04 Read and write numerals to 30 and connect them to the quantities they represent.
- N.ME.00.05 Count orally to 100 by ones. Count to 30 by 2's, 5's and 10's using grouped objects as needed.

EL.4. Develop strategies for estimating quantity and evaluate the reasonableness of their estimates.

Michigan Content Standard:

3. Number Relationships

- Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Grade Level Benchmark:

EL.1. Compare and order numbers using “equal,” “less than” or “greater than.”

Grade Level Content Expectation:

- N.ME.00.02 Use one-to-one correspondence to compare and order sets of objects to 30 using such phrases as “same number”, “more than”, or “less than”; use counting and matching.
- N.ME.00.03 Compare and order numbers to 30 using phrases such as “more than” or “less than.”

EL.2. Use part-whole relationships to explore numbers, develop number concepts and understand computation.

Grade Level Content Expectation:

- N.ME.01.08 List number facts (partners inside of numbers) for 2 through 10; e.g., $8 = 7 + 1 = 6 + 2 = 5 + 3 = 4 + 4$; $10 = 8 + 2 = 2 + 8$.

EL.3. Classify numbers as even or odd and explore concepts of factors and multiples.

Strand V: Numerical and Algebraic Operations and Analytical Thinking

Michigan Content Standard:

1. Operations and their Properties

- Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.

Grade Level Benchmark:

EL.1. Use manipulatives to model operations with numbers; develop their own methods of recording operations; and relate their models and recordings to standard symbolic expressions and algorithms.

Grade Level Content Expectation:

- N.MR.00.08 Describe and make drawings to represent situations/stories involving putting together and taking apart for totals up to 10; use finger and object counting.
- N.MR.00.09 Record mathematical thinking by writing simple addition and subtraction sentences, e.g., $7 + 2 = 9$, $10 - 8 = 2$.

EL.2. Develop and apply the appropriate method of computation from among mental computation, estimation, paper-and-pencil or calculators; explain why they are choosing a method and how they know which operations to perform in a given situation.

Michigan Content Standard:

2. Algebraic and Analytic Thinking

- Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Grade Level Benchmark:

1. Write and solve open sentences (e.g., $+ = 5$) and write stories to fit the open sentence.

Grade Level Content Expectation:

- N.MR.00.08 Describe and make drawings to represent situations/stories involving putting together and taking apart for totals up to 10; use finger and object counting.

EL.2. Explore algebraic concepts with manipulatives such as balance scales, tables of input and output, and pictorial representations of problems.

EL.3. Find replacements for the variable(s) in open sentences.

Strand VI: Probability and Discrete Mathematics

Michigan Content Standard:

2. Discrete Mathematics

- Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.

Grade Level Benchmark:

EL.1. Use manipulatives and diagrams to explore problems involving counting and arranging objects.

EL.2. Explore sets and set relationships by sorting and classifying objects.

Grade Level Content Expectation:

- G.GS.00.02 Identify, sort and classify objects by attribute and identify objects that do not belong in a particular group.

EL.4. Explore now-next patterns.

EL.5. Explore, develop and invent their own algorithms to accomplish a task or to solve numerical problems.

Mathematics Revision 2006
Scope and Sequence by Grade
Grade One

Strand I – Patterns, Relationships, Functions

Michigan Content Standard:

1. Patterns:

- Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.

Grade Level Benchmark:

EL.1. Recognize, describe and extend numerical and geometric patterns.

EL.2. Represent and record patterns and relationships in a variety of ways including tables, charts and pictures.

Grade Level Content Expectation:

- G.SR.01.03 Create and describe patterns, such as repeating patterns, and growing patterns using number, shape, and size.

EL.3. Use patterns to describe real-world phenomena.

EL.4. Explore various types of numeric and geometric patterns (repeating, growing, shrinking).

Grade Level Content Expectation:

- G.SR.01.03 Create and describe patterns, such as repeating patterns, and growing patterns using number, shape, and size.
- G.SR.01.05 Predict the next element in a simple repeating pattern.
- G.SR.01.06 Describe ways to get to the next element in simple repeating patterns.

EL.5. Apply their experiences with patterns to help solve problems and explore new content.

Grade Level Content Expectation:

- G.SR.01.05 Predict the next element in a simple repeating pattern.

Michigan Content Standard:

2. Variability and Change:

- Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

Grade Level Benchmark:

EL.1. Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

EL.2. Recognize that change is often predictable, but variable, and that patterns emerge that help to describe the change.

EL.3. Explore change, and realize that changes are frequently interdependent.

EL.4. Use tables, charts, open sentences, and hands-on models to represent change and variability.

EL.5. Explore change, and realize that changes are frequently interdependent.

Strand II: Geometry and Measurement

Michigan Content Standard:

1. Shape and Shape Relationship

- Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

Grade Level Benchmark:

EL.4. Draw and build familiar shapes.

Grade Level Content Expectation:

- G.GS.01.01 Create common two-dimensional and three-dimensional shapes and describe their physical and geometric attributes, such as color and shape.

EL.7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.

Michigan Content Standard:

2. Position

- Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

Grade Level Benchmark:

EL1. Locate and describe objects in terms of their position, including front, back, inside, outside, right, left, over, under, next to, between and locations on the number line on a coordinate graph and on a map.

Grade Level Content Expectation:

- N.ME.01.05 Understand that a number to the right of another number on the number line is bigger and that a number to the left is smaller.
- G.LO.01.02 Describe relative position of objects on a plane and in space, using words such as above, below, behind, in front of.

EL.2. Locate and describe objects in terms of their orientation, direction and relative position, including up, down, back, N-S-E-W, flipped, turned, translated; recognize symmetrical objects and identify their lines of symmetry.

Grade Level Content Expectation:

- G.LO.01.02 Describe relative position objects on a plane and in space, using words such as above, below, behind, in front of.

EL.3. Explore what happens to the size, shape and position of an object after sliding, flipping, turning, enlarging or reducing it.

EL.5. Use concepts of position, direction, and orientation to describe the physical; world and to solve problems.

Grade Level Content Expectation:

- G.LO.01.02 Describe relative position of objects on a plane and in space, using words such as above, below, behind, in front of.

Michigan Content Standard:

3. Measurement

- Students compare attributes of two objects, or of one with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

Grade Level Benchmark:

EL.1. Compare attributes of objects; develop standard units of measurement; and select and use standard tools for measurement.

Grade Level Content Expectation:

- M.UN.01.01 Measure the lengths of objects in non-standard units (e.g., pencil, lengths, shoe lengths) to the nearest whole unit.
- M.UN.01.02 Compare measured lengths using the words shorter, shortest, longer, longest, taller, tallest, etc.
- M.UN.01.03 Tell time on a twelve-hour clock face to the hour and half hour. Work with money.
- M.UN.01.04 Identify the different denominations of coins and bills.
- M.UN.01.05 Match one coin or bill of one denomination to an equivalent set of coins/bills of other denominations, e.g., 1 quarter = 2 dimes and 1 nickel.
- M.UN.01.06 Tell the amount of money: in cents up to \$1, in dollars up to \$100. Use the symbols \$ and ¢.
- M.PS.01.07 Add and subtract money in dollars only or in cents only.

EL.2. Identify the attribute to be measured and select the appropriate unit of measurement for length, mass, weight, area, perimeter, capacity, time, temperature and money.

Grade Level Content Expectation:

- M.UN.01.06 Tell the amount of money: in cents up to \$1, in dollars up to \$100. Use the symbols \$ and ¢.

EL.3. Develop strategies for estimating measures and compare the estimates to the results of the measurement; decide if an estimate is “a good estimate.”

Grade Level Content Expectation:

- M.UN.01.01 Measure the lengths of objects in non-standard units (e.g., pencil lengths, shoe lengths) to the nearest whole unit.
- M.UN.01.02 Compare measured lengths using the words shorter, shortest, longer, longest, taller, tallest, etc.

EL.4. Explain the meaning of measurements and recognize that the number of units it takes to measure an object is related to the size of the unit.

Grade Level Content Expectation:

- M.UN.01.01 Measure the lengths of objects in non-standard units (e.g., pencil lengths, shoe lengths) to the nearest whole unit.
- M.UN.01.03 Tell time on a twelve-hour clock face to the hour and half hour.

EL.6. Apply measurement to describe the real world and to solve problems.

Strand III: Data Analysis and Statistics

Michigan Content Standard:

1. Collection, Organization, Presentation of Data

- Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.

Grade Level Benchmark:

EL.1. Collect and explore data through counting, measuring, and conducting surveys and experiments.

Grade Level Content Expectation:

- D.RE.01.01 Collect and organize data to use in pictographs.

EL.2. Organize data using concrete objects, pictures, tallies, tables, charts, diagrams, and graphs.

Grade Level Content Expectation:

- D.RE.01.01 Collect and organize data to use in pictographs.
- D.RE.01.03 Make pictographs of given data, using both horizontal and vertical forms of graphs; scale should be in units of one and include symbolic representations, e.g., represents one child.

EL.3. Present data using a variety of appropriate representations and explain the meaning of the data.

Grade Level Content Expectation:

- D.RE.01.03 Make pictographs of given data, using both horizontal and vertical forms of graphs; scale should be in units one and include symbolic representations, e.g., represents one child.

Michigan Content Standard:

2. Description and Interpretations

- Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.

Grade Level Benchmark:

EL.1. Read and explain data they have collected and organized themselves and progress to reading data from other sources.

Grade Level Content Expectation:

- D.RE.01.01 Collect and organize data to use in pictographs.
- D.RE.01.02 Read and interpret pictographs.
- D.RE.01.03 Make pictographs of given data, using both horizontal and vertical forms of graphs; scale should be in units of one and include symbolic representations, e.g., represents one child.

EL.2. Describe the shape of the data using informal language.

EL.3. Draw, explain, and justify conclusions, such as trends based on data.

EL.5. Formulate questions and problems and gather and interpret data to answer those questions.

Michigan Content Standard:

3. Inference and Prediction

- Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.

Grade Level Benchmark:

EL.1. Make and test hypotheses.

EL.3. Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.

Strand IV: Number Sense and Numeration

Michigan Content Standard:

1. Concepts and Properties of Numbers

- Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.

Grade Level Benchmark:

EL.1. Develop an understanding of whole numbers; and read, write and count using whole numbers; investigate basic concepts of fractions and decimals.

Grade Level Content Expectation:

- N.ME.01.01 Count to 110 by 1's, 2's, 5's, and 10's, starting from any number in the sequence; count to 500 by 100's and 10's; use ordinals to identify position in a sequence, e.g., 1st, 2nd, 3rd.
- N.ME.01.02 Read and write numbers to 110 and relate them to the quantities they represent.
- N.ME.01.03 Order numbers to 110; compare using the phrases: same as, more than, greater than, fewer than; use = symbol. Arrange small sets of numbers in increasing or decreasing order, e.g., write the following from smallest to largest: 21, 16, 35, 8.
- N.ME.01.04 Identify one more than, one less than, 10 more than, and 10 less than for any number up to 100.
- N.ME.01.05 Understand that a number to the right of another number on the number line is bigger and that a number to the left is smaller.
- N.ME.01.06 Count backward by 1's starting from any number between 1 and 100.
- N.ME.01.07 Compose and decompose numbers to 30, including using bundles of tens and units, e.g., recognize 24 as 2 tens and 4 ones, 10 and 10 and 4, 20 and 4, and 24 ones.

EL.2. Investigate and develop an understanding of the base 10, place-value system.

Grade Level Content Expectation:

- N.ME.01.01 Count to 110 by 1's, 2's, 5's, and 10's, starting from any number in the sequence; count to 500 by 100's and 10's; use ordinals to identify position in a sequence, e.g., 1st, 2nd, 3rd.
- N.ME.01.02 Read and write numbers to 110 and relate them to the quantities they represent.

- N.ME.01.04 Identify one more than, one less than, 10 more than, and 10 less than for any number up to 100.
- N.ME.01.06 Count backward by 1's starting from any number between 1 and 100.
- N.ME.01.07 Compose and decompose numbers to 30, including using bundles of tens and units, e.g., recognize 24 as 2 tens and 4 ones, 10 and 10 and 4, 20 and 4, and 24 ones.

EL.3. Develop an understanding of the properties of numbers (e.g., order) and of the properties of the special numbers 0 and 1.

Grade Level Content Expectation:

- N.ME.01.07 Compose and decompose numbers to 30, including using bundles of tens and units, e.g., recognize 24 as 2 tens and 4 ones, 10 and 10 and 4, 20 and 4, and 24 ones.

EL.4. Apply their understanding of number systems to model and solve problems.

Michigan Content Standard:

2. Representation and Uses of Numbers

- Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Grade Level Benchmark:

EL.1. Represent whole numbers, fractions, and decimals using concrete, pictorial and symbolic representations.

EL.2. Explore and recognize different representations for the same number and explain why they are the same.

Grade Level Content Expectation:

- N.ME.01.03 Order numbers to 110; compare using the phrases: same as, more than, greater than, fewer than; use = symbol. Arrange small sets of numbers in increasing or decreasing order, e.g., write the following from smallest to largest: 21, 16, 35, 8.
- N.ME.01.07 Compose and decompose numbers to 30, including using bundles of tens and units, e.g., recognize 24 as 2 tens and 4 ones, 10 and 10 and 4, 20 and 4, and 24 ones.

EL.3. Investigate ways numbers are used (e.g., counting, ordering, naming, locating, measuring).

Grade Level Content Expectation:

- N.ME.01.01 Count to 110 by 1's, 2's, 5's and 10's, starting from any number in the sequence; count to 500 by 100's and 10's; use ordinals to identify position in a sequence, e.g., 1st, 2nd, 3rd.
- N.ME.01.02 Read and write numbers to 110 and relate them to the quantities they represent.
- N.ME.01.03 Order numbers to 110; compare using the phrases: same as, more than, greater than, fewer than; use = symbol. Arrange small sets of numbers in increasing or decreasing order, e.g., write the following from smallest to largest: 21, 16, 35, 8.

EL.4. Develop strategies for estimating quantity and evaluate the reasonableness of their estimates.

EL.5. Select appropriate numbers and representations in order to solve problems.

Michigan Content Standard:

3. Number Relationships

Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Grade Level Benchmark:

EL.1. Compare and order numbers using “equal,” “less than” or “greater than.”

Grade Level Content Expectation:

- ME.01.03 Order numbers to 110; compare using the phrases: same as, more than, greater than, fewer than; use = symbol. Arrange small sets of numbers in increasing or decreasing order, e.g., write the following from smallest to largest: 21, 16, 35, 8.
- N.ME.01.04 Identify one more than, one less than, 10 more than, and 10 less than for any number up to 100.
- N.ME.01.05 Understand that a number to the right of another number on the number line is bigger and that a number to the left is smaller.

EL.2. Use part-whole relationships to explore numbers, develop number concepts and understand computation.

Grade Level Content Expectation:

- N.ME.01.07 Compose and decompose numbers to 30, including using bundles of tens and units, e.g., recognize 24 as 2 tens and 4 ones, 10 and 10 and 4, 20 and 4, and 24 ones.
- N.ME.01.08 List number facts (partners inside of numbers) for 2 through 10; e.g., $8 = 7 + 1 = 6 + 2 = 5 + 3 = 4 + 4$; $10 = 8 + 2 = 2 + 8$.

EL.3. Classify numbers as even or odd and explore concepts of factors and multiples.

EL.5. Apply their understanding of number relationships in solving problems.

Strand V: Numerical and Algebraic Operations and Analytical Thinking

Michigan Content Standard:

1. Operations and their Properties

- Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.

Grade Level Benchmark:

EL.1. Use manipulatives to model operations with numbers; develop their own methods of recording operations; and relate their models and recordings to standard symbolic expressions and algorithm.

Grade Level Content Expectation:

- N.MR.01.10 Model addition and subtraction for numbers less than 20 for a given contextual situation using objects or pictures; explain in words; record using numbers and symbols; solve.

EL.2. Develop and apply the appropriate method of computation from among mental computation, estimation, paper-and-pencil or calculators; explain why they are choosing a method and how they know which operations to perform in a given situation.

Grade Level Content Expectation:

- N.FL.01.12 Know all the addition facts up to $10 + 10$, and solve the related subtraction problems fluently.
- N.FL.01.14 Add three one-digit numbers.
- N.FL.01.15 Calculate mentally sums and differences involving: a two-digit number and a one-digit number without regrouping; a two-digit number and a multiple of 10.
- N.FL.01.16 Compute sums and differences up to two-digit numbers using number facts and strategies, but no formal algorithm.

EL.3. Explore properties of operations (e.g., commutative and distributive properties) and give examples of how they use those properties.

Grade Level Content Expectation:

- N.ME.01.07 Compose and decompose numbers to 30, including using bundles of tens and units, e.g., recognize 24 as 2 tens and 4 ones, 10 and 10 and 4, 20 and 4, and 24 ones.
- N.MR.01.11 Understand the inverse relationship between addition and subtraction, e.g., subtraction “undoes” addition: if $3 + 5 = 8$, we know that $8 - 3 = 5$ and $8 - 5 = 3$; recognize that some problems involving combining, “taking away”, or comparing can be solved by either operation.

- N.FL.01.12 Know all the addition facts up to $10 + 10$, and solve the related subtraction problems fluently.

EL.4. Apply operations efficiently and accurately in solving problems.

Michigan Content Standard:

2. Algebraic and Analytic Thinking

- Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Grade Level Benchmark:

EL.1. Write and solve open sentences (e.g., $+ = 5$) and write stories to fit the open sentence.

Grade Level Content Expectation:

- N.MR.01.13 Apply knowledge of fact families to solve simple open sentences for addition and subtraction, such as: $+ 2 = 7$ and $10 - = 6$.

EL.2. Explore algebraic concepts with manipulatives such as balance scales, tables of input and output, and pictorial representations of problems.

Grade Level Content Expectation:

- N.MR.01.10 Model addition and subtraction for numbers less than 20 for a given contextual situation using objects or pictures; explain in words; record using numbers and symbols; solve.

EL.3. Find replacements for the variable(s) in open sentences.

Grade Level Content Expectation:

- N.MR.01.13 Apply knowledge of fact families to solve simple open sentences for addition and subtraction, such as: $+ 2 = 7$ and $10 - = 6$.

Strand VI: Probability and Discrete Mathematics

Michigan Content Standard:

2. Discrete Mathematics

- Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.

Grade Level Benchmark:

EL.1. Use manipulatives and diagrams to explore problems involving counting and arranging objects.

EL.2. Explore sets and set relationships by sorting and classifying objects.

Grade Level Content Expectation:

- N.ME.01.03 Order numbers to 110;compare using the phrases: same as, more than, greater than, fewer than; use = symbol. Arrange small sets of numbers in increasing or decreasing order, e.g., write the following from smallest to largest: 21, 16, 35, 8.
- N.MR.01.09 Compare two or more sets in terms of the difference in number of elements.

EL.4. Explore now-next patterns.

EL.5. Explore, develop and invent their own algorithms to accomplish a task or to solve numerical problems.

Grade Level Content Expectation:

- N.FL.01.16 Compute sums and differences up to two-digit numbers using number facts and strategies, but no formal algorithm.

Mathematics Revision 2006
Scope and Sequence by Grade
Grade Two

Strand I – Patterns, Relationships, Functions

Michigan Content Standard:

1. Patterns:

- Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.

Grade Level Benchmark:

EL. 1. Recognize, describe and extend numerical and geometric patterns.

Grade Level Content Expectation:

- N.ME.02.01 Count to 1000 by 1's, 10's, and 100's starting from any number in the sequence.
- N.ME.02.04 Count orally by 3's and 4's starting with 0, and by 2's, 5's, and 10's starting from any number.

EL. 2. Represent and record patterns and relationships in a variety of ways including tables, charts and pictures.

EL. 3. Use patterns to describe real-world phenomena.

EL. 4. Explore various types of numeric and geometric patterns (repeating, growing, shrinking).

EL. 5. Apply their experiences with patterns to help solve problems and explore new content.

Michigan Content Standard:

2. Variability and Change:

- Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

Grade Level Benchmark:

EL. 4. Use tables, charts, open sentences, and hands-on models to represent change and variability.

EL. 5. Begin to describe and differentiate between types of relationships, especially repeating, growing and shrinking patterns.

Strand II: Geometry and Measurement

Michigan Content Standard:

1. Shape and Shape Relationship

- Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

Grade Level Benchmark:

EL. 1. Recognize and name familiar shapes in one, two and three dimensions such as lines, rectangles and spheres and informally discuss the shape of a graph.

Grade Level Content Expectation:

- G.GS.02.01 Identify, describe, and compare familiar two-dimensional and three-dimensional shapes, such as triangles, rectangles, squares, circles, semi-circles, spheres, rectangular prisms.

EL. 2. Describe the attributes of familiar shapes.

Grade Level Content Expectation:

- G.GS.02.01 Identify, describe, and compare familiar two-dimensional and three-dimensional shapes, such as triangles, rectangles, squares, circles, semi-circles, spheres, rectangular prisms.
- G.GS.02.04 Distinguish between curves and straight lines and between curved surfaces and flat surfaces.
- G.SR.02.05 Classify familiar plane and solid objects, e.g., square, rectangle, rhombus, cube, pyramid, prism, cone, cylinder, and sphere, by common attributes such as shape, size, color, roundness or number of corners and explain which attributes are being used for classification.

EL. 3. Compare, sort, and classify familiar shapes.

Grade Level Content Expectation:

- G.GS.02.01 Identify, describe, and compare familiar two-dimensional and three-dimensional shapes, such as triangles, rectangles, squares, circles, semi-circles, spheres, rectangular prisms.
- G.GS.02.02 Explore and predict the results of putting together and taking apart twodimensional and three-dimensional shapes.

- G.SR.02.05 Classify familiar plane and solid objects, e.g., square, rectangle, rhombus, cube, pyramid, prism, cone, cylinder, and sphere, by common attributes such as shape, size, color, roundness or number of corners and explain which attributes are being used for classification.

EL. 4. Draw and build familiar shapes.

Grade Level Content Expectation:

- G.GS.02.02 Explore and predict the results of putting together and taking apart two-dimensional and three-dimensional shapes.
- G.GS.02.03 Draw rectangles and triangles, and compute perimeters by adding lengths of sides, recognizing the meaning of perimeter.

EL. 5. Explore ways to combine, dissect and transform shapes.

Grade Level Content Expectation:

- G.GS.02.02 Explore and predict the results of putting together and taking apart twodimensional and three-dimensional shapes.

EL. 6. Recognize parallel and perpendicular line segments and figures that have similarity and/or congruence.

EL. 7. Use shape, shape properties, and shape relationships to describe the physical world and to solve problems.

Grade Level Content Expectation:

- G.GS.02.01 Identify, describe, and compare familiar two-dimensional and three-dimensional shapes, such as triangles, rectangles, squares, circles, semi-circles, spheres, rectangular prisms.
- G.GS.02.02 Explore and predict the results of putting together and taking apart twodimensional and three-dimensional shapes.
- G.GS.02.03 Draw rectangles and triangles, and compute perimeters by adding lengths of sides, recognizing the meaning of perimeter.
- G.SR.02.05 Classify familiar plane and solid objects, e.g., square, rectangle, rhombus, cube, pyramid, prism, cone, cylinder, and sphere, by common attributes such as shape, size, color, roundness or number of corners and explain which attributes are being used for classification.

Strand II: Geometry and Measurement

Michigan Content Standard:

2. Position

- Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

Grade Level Benchmark:

EL. 1. Locate and describe objects in terms of their position, including front, back, inside, outside, right, left, over, under, next to, between and locations on the number line on a coordinate graph and on a map.

Grade Level Content Expectation:

- N.ME.02.20 Place 0 and halves, e.g., $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$ on the number line; relate to a ruler.
- G.LO.02.07 Find and name locations using simple coordinate systems such as maps and first quadrant grids.

EL. 2. Locate and describe objects in terms of their orientation, direction and relative position, including up, down, back, N-S-E-W, flipped, turned, translated; recognize symmetrical objects and identify their lines of symmetry.

Grade Level Content Expectation:

- G.LO.02.07 Find and name locations using simple coordinate systems such as maps and first quadrant grids.

EL. 3. Explore what happens to the size, shape, and position of an object after sliding, flipping, turning, enlarging or reducing it.

Grade Level Content Expectation:

- G.TR.02.06 Recognize that shapes that have been slid, turned or flipped are the same shape, e.g., a square rotated 45 degrees is still a square.

EL. 4. Use concepts of position, direction and orientation to describe the physical; world and to solve problems.

Grade Level Content Expectation:

- G.TR.02.06 Recognize that shapes that have been slid, turned or flipped are the same shape, e.g., a square rotated 45° is still a square.
- G.LO.02.07 Find and name locations using simple coordinate systems such as maps and first quadrant grids.

Michigan Content Standard:

3. Measurement

- Students compare attributes of two objects, or of one with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

Grade Level Benchmark:

EL. 1. Compare attributes of objects; develop standard units of measurement; and select and use standard tools for measurement.

Grade Level Content Expectation:

- M.UN.02.01 Measure lengths in meters, centimeters, inches, feet, and yards approximating to the nearest whole unit and using abbreviations: cm, m, in, ft, yd.
- M.UN.02.05 Using both A.M. and P.M., tell and write time from the clock face in 5 minute intervals, and from digital clocks to the minute; include reading time: 9:15 as nine-fifteen and 9:50 as nine-fifty. Interpret time both as minutes after the hour and minutes before the next hour, e.g., 8:50 as eight-fifty and ten to nine. Show times by drawing hands on clock face.
- M.UN.02.06 Use the concept of duration of time, e.g., determine what time it will be half an hour from 10:15.
- M.UN.02.07 Read and write amounts of money using decimal notations, e.g., \$1.15.
- M.PS.02.08 Add and subtract money in mixed units, e.g., \$2.50 + 60 cents and \$5.75 - \$3, but not \$2.50 + \$3.10.
- M.UN.02.09 Read temperature using the scale on a thermometer in degrees Fahrenheit.

EL. 2. Identify the attribute to be measured and select the appropriate unit of measurement for length, mass, weight, area, perimeter, capacity, time, temperature, and money.

Grade Level Content Expectation:

- M.UN.02.05 Using both A.M. and P.M., tell and write time from the clock face in minute intervals, and from digital clocks to the minute; include reading time: 9:15 as nine-fifteen and 9:50 as nine-fifty. Interpret time both as minutes after the hour and minutes before the next hour, e.g., 8:50 as eight-fifty and ten to nine. Show times by drawing hands on clock face.
- M.UN.02.06 Use the concept of duration of time, e.g., determine what time it will be half an hour from 10:15.
- M.UN.02.07 Read and write amounts of money using decimal notations, e.g., \$1.15.

EL. 3. Develop strategies for estimating measures and compare the estimates to the results of the measurement; decide if an estimate is “a good estimate.”

Grade Level Content Expectation:

- M.UN.02.01 Measure lengths in meters, centimeters, inches, feet, and yards approximating to the nearest whole unit and using abbreviations: cm, m, in, ft, yd.
- M.UN.02.03 Measure area using nonstandard units to the nearest whole unit.

EL. 4. Explain the meaning of measurements and recognize that the number of units it takes to measure an object is related to the size of the unit.

Grade Level Content Expectation:

- M.UN.02.01 Measure lengths in meters, centimeters, inches, feet, and yards approximating to the nearest whole unit and using abbreviations: cm, m, in, ft, yd.
- M.UN.02.03 Measure area using nonstandard units to the nearest whole unit.
- M.TE.02.04 Find the area of a rectangle with whole number side lengths by covering with unit squares and counting, or by using a grid of unit squares; write the area as a product.

EL. 5. Explore scale drawings, models, and maps and relate them to measurements of real objects.

EL. 6. Apply measurement to describe the real world and to solve problems.

Grade Level Content Expectation:

- M.PS.02.02 Compare lengths; add and subtract lengths (no conversion of units).
- M.PS.02.08 Add and subtract money in mixed units, e.g., \$2.50 + 60 cents and \$5.75 - \$3, but not \$2.50 + \$3.10.
- M.PS.02.10 Solve simple word problems involving length and money.

Strand III: Data Analysis and Statistics

Michigan Content Standard:

1. Collection, Organization, Presentation of Data

- Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.

Grade Level Benchmark:

EL. 1. Collect and explore data through counting, measuring and conducting surveys and experiments.

EL. 2. Organize data using concrete objects, pictures, tallies, tables, charts, diagrams and graphs.

Grade Level Content Expectation:

- D.RE.02.01 Make pictographs using a scale representation, using scales where symbols equal more than one.

EL. 3. Present data using a variety of appropriate representations and explain the meaning of the data.

Michigan Content Standard:

2. Description and Interpretations

- Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.

Grade Level Benchmark:

EL. 1. Read and explain data they have collected and organized themselves and progress to reading data from other sources.

Grade Level Content Expectation:

- D.RE.02.02 Read and interpret pictographs with scales, using scale factors of 2 and 3.

EL. 2. Describe the shape of the data using informal language.

EL. 3. Draw, explain and justify conclusions, such as trends based on data.

Grade Level Content Expectation:

- D.RE.02.03 Solve problems using Information in pictographs; include scales such as “each ■ represents 2 apples”; avoid partial cases.

EL. 4. Raise and answer questions about the source, collection, organization and presentation of data, as well as the conclusions drawn from the data; explore biases in data.

EL. 5. Formulate questions and problems and gather and interpret data to answer those questions.

Grade Level Content Expectation:

- D.RE.02.03 Solve problems using. Information in pictographs; include scales such as “each ■ represents 2 apples”; avoid partial cases.

Michigan Content Standard:

3. Inference and Prediction

- Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.

Grade Level Benchmark:

EL. 1. Make and test hypotheses.

EL. 3. Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.

EL. 5. Make predictions to answer questions and solve problems.

Strand IV: Number Sense and Numeration

Michigan Content Standard:

1. Concepts and Properties of Numbers

- Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.

Grade Level Benchmark:

EL. 1. Develop an understanding of whole numbers; and read, write and count using whole numbers; investigate basic concepts of fractions and decimals.

Grade Level Content Expectation:

- N.ME.02.01 Count to 1000 by 1's, 10's, and 100's starting from any number in the sequence.
- N.ME.02.02 Read and write numbers to 1000 in numerals and words, and relate them to the quantities they represent.
- N.ME.02.03 Compare and order numbers to 1000; use the symbols $>$ and $<$.
- N.ME.02.04 Count orally by 3's and 4's starting with 0, and by 2's, 5's, and 10's starting from any number.
- N.ME.02.05 Express numbers up to 1000 using place value, e.g., 137 is 1 hundred, 3 tens, and 7 ones; use concrete materials.
- N.ME.02.18 Recognize, name, and represent commonly used unit fractions with denominators 12 or less; model $\frac{1}{2}$, and $\frac{1}{3}$, $\frac{1}{4}$ by folding strips.
- N.ME.02.19 Recognize, name, and write commonly used fractions: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$.
- N.ME.02.20 Place 0 and halves, e.g., $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$ on the number line; relate to a ruler.
- N.ME.02.21 For unit fractions from $\frac{1}{12}$ to $\frac{1}{2}$, understand the inverse relationship between the size of a unit fraction and the size of the denominator; compare unit fractions from $\frac{1}{12}$ to $\frac{1}{2}$.
- N.ME.02.22 Recognize that fractions such as $\frac{2}{2}$ and $\frac{3}{3}$, $\frac{2}{2}$ are equal to the whole (one).

EL. 2. Investigate and develop an understanding of the base 10, place-value system.

Grade Level Content Expectation:

- N.ME.02.02 Read and write numbers to 1000 in numerals and words, and relate them to the quantities they represent.

- N.ME.02.03 Compare and order numbers to 1000; use the symbols $>$ and $<$.
- N.ME.02.05 Express numbers up to 1000 using place value, e.g., 137 is 1 hundred, 3 tens, and 7 ones; use concrete materials.

EL. 3. Develop an understanding of the properties of numbers (e.g., order) and of the properties of the special numbers 0 and 1.

Grade Level Content Expectation:

- N.ME.02.05 Express numbers up to 1000 using place value, e.g., 137 is 1 hundred, 3 tens, and 7 ones; use concrete materials.

EL. 4. Apply their understanding of number systems to model and solve problems.

Michigan Content Standard:

2. Representation and Uses of Numbers

- Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Grade Level Benchmark:

EL. 1. Represent whole numbers, fractions and decimals using concrete, pictorial and symbolic representations.

Grade Level Content Expectation:

- N.ME.02.05 Express numbers up to 1000 using place value, e.g., 137 is 1 hundred, 3 tens, and 7 ones; use concrete materials.
- N.ME.02.18 Recognize, name, and represent commonly used unit fractions with denominators 12 or less; model $\frac{1}{2}$, and $\frac{1}{3}$, $\frac{1}{4}$ by folding strips.
- N.ME.02.19 Recognize, name, and write commonly used fractions: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$.
- N.ME.02.20 Place 0 and halves, e.g., $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$ on the number line; relate to a ruler.

EL. 2. Explore and recognize different representations for the same number and explain why they are the same.

Grade Level Content Expectation:

- N.FL.02.06 Decompose 100 into addition pairs, e.g., $99 + 1$, $98 + 2$...
- N.ME.02.18 Recognize, name, and represent commonly used unit fractions with denominators 12 or less; model $\frac{1}{2}$, and $\frac{1}{3}$, $\frac{1}{4}$ by folding strips.
- N.ME.02.19 Recognize, name, and write commonly used fractions: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$.

- N.ME.02.22 Recognize that fractions such as $\frac{4}{4}$ and $\frac{3}{3}$, $\frac{2}{3}$ are equal to the whole (one).
- N.MR.02.13 Understand multiplication as the result of counting the total number of objects in a set of equal groups, e.g., 3×5 gives the number of objects in 3 groups of 5 objects, or $3 \times 5 = 5 + 5 + 5 = 15$.

EL. 3. Investigate ways numbers are used (e.g., counting, ordering, naming, locating, measuring).

Grade Level Content Expectation:

- N.ME.02.01 Count to 1000 by 1's, 10's, and 100's starting from any number in the sequence.
- N.ME.02.02 Read and write numbers to 1000 in numerals and words, and relate them to the quantities they represent.
- N.ME.02.03 Compare and order numbers to 1000; use the symbols $>$ and $<$.
- N.ME.02.04 Count orally by 3's and 4's starting with 0, and by 2's, 5's, and 10's starting from any number.

EL. 4. Develop strategies for estimating quantity and evaluate the reasonableness of their estimates.

Grade Level Content Expectation:

- N.FL.02.11 Estimate and calculate the sum of two numbers with three digits that do not require regrouping.

Michigan Content Standard:

3. Number Relationships

- Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Grade Level Benchmark:

EL. 1. Compare and order numbers using “equal,” “less than” or “greater than.”

Grade Level Content Expectation:

- N.ME.02.03 Compare and order numbers to 1000; use the symbols $>$ and $<$.
- N.ME.02.20 Place 0 and halves, e.g., $\frac{2}{1}$, $\frac{2}{2}$, $\frac{1}{1}$, $\frac{2}{1}$ on the number line; relate to a ruler.

EL. 2. Use part-whole relationships to explore numbers, develop number concepts and understand computation.

Grade Level Content Expectation:

- N.ME.02.05 Express numbers up to 1000 using place value, e.g., 137 is 1 hundred, 3 tens, and 7 ones; use concrete materials.
- N.MR.02.13 Understand multiplication as the result of counting the total number of objects in a set of equal groups, e.g., 3×5 gives the number of objects in 3 groups of 5 objects, or $3 \times 5 = 5 + 5 + 5 = 15$.
- N.ME.02.22 Recognize that fractions such as $\frac{4}{4}$ and $\frac{3}{3}$, $\frac{2}{2}$ are equal to the whole (one).

EL. 3. Classify numbers as even or odd and explore concepts of factors and multiples.

Strand V: Numerical and Algebraic Operations and Analytical Thinking

Michigan Content Standard:

1. Operations and their Properties

- Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.

Grade Level Benchmark:

EL. 1. Use manipulatives to model operations with numbers; develop their own methods of recording operations; and relate their models and recordings to standard symbolic expressions and algorithms.

Grade Level Content Expectation:

- N.MR.02.07 Find the distance between numbers on the number line, e.g., how far is 79 from 26?
- N.MR.02.09 Given a contextual situation that involves addition and subtraction for numbers up to two digits: model using objects or pictures, explain in words, record using numbers and symbols; solve.
- N.MR.02.13 Understand multiplication as the result of counting the total number of objects in a set of equal groups, e.g., 3×5 gives the number of objects in 3 groups of 5 objects, or $3 \times 5 = 5 + 5 + 5 = 15$.
- N.MR.02.14 Represent multiplication using area and array models.
- N.MR.02.16 Given a simple situation involving groups of equal size or of sharing equally, represent with objects, words, and symbols; solve.

EL. 2. Develop and apply the appropriate method of computation from among mental computation, estimation, paper-and-pencil or calculators; explain why they are choosing a method and how they know which operations to perform in a given situation.

Grade Level Content Expectation:

- N.FL.02.10 Add fluently two numbers up to two digits each, using strategies including formal algorithms; subtract fluently two numbers up to two digits each.
- N.FL.02.12 Calculate mentally sums and differences involving: three-digit numbers and ones; three-digit numbers and tens; three-digit numbers and hundreds.
- N.FL.02.17 Develop strategies for fluently multiplying numbers up to 5×5 .

EL. 3. Explore properties of operations (e.g., commutative and distributive properties) and give examples of how they use those properties.

Grade Level Content Expectation:

- N.ME.02.05 Express numbers up to 1000 using place value, e.g., 137 is 1 hundred, 3 tens, and 7 ones; use concrete materials.
- N.MR.02.15 Understand division (\div) as another way of expressing multiplication, using fact families within the 5×5 multiplication table; emphasize that division “undoes” multiplication, e.g., $2 \times 3 = 6$ can be rewritten as $6 \div 2 = 3$ or $6 \div 3 = 2$.

EL. 4. Apply operations efficiently and accurately in solving problems.

Michigan Content Standard:

2. Algebraic and Analytic Thinking

- Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Grade Level Benchmark:

EL. 1. Write and solve open sentences (e.g., $\square + = 5$) and write stories to fit the open sentence.

Grade Level Content Expectation:

- N.MR.02.08 Find missing values in open sentences, e.g., $42 + = 57$; use relationship between addition and subtraction.

EL. 2. Explore algebraic concepts with manipulatives such as balance scales, tables of input and output, and pictorial representations of problems.

Grade Level Content Expectation:

- N.MR.02.09 Given a contextual situation that involves addition and subtraction for numbers up to two digits: model using objects or pictures, explain in words, record using numbers and symbols; solve.
- D.RE.02.01 Make pictographs using a scale representation, using scales where symbols equal more than one.

EL. 3. Find replacements for the variable(s) in open sentences.

Grade Level Content Expectation:

- N.MR.02.08 Find missing values in open sentences, e.g., $42 + \square = 57$; use relationship between addition and subtraction.

Strand VI: Probability and Discrete Mathematics

Michigan Content Standard:

1. Probability

- Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations.

Grade Level Benchmark:

EL. 1. Explain the difference between chance and certainty and give examples to illustrate their understanding.

EL. 2. Compare events and describe them as “more likely” or “less likely” and use the language of fractions to describe simple probabilities.

EL. 3. Conduct experiments with concrete objects to explore concepts and develop an intuitive understanding of how the conditions of the experiment can affect the outcome.

Michigan Content Standard:

2. Discrete Mathematics

Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.

Grade Level Benchmark:

EL. 1. Use manipulatives and diagrams to explore problems involving counting and arranging objects.

EL. 2. Explore sets and set relationships by sorting and classifying objects.

EL. 4. Explore now-next patterns.

EL. 5. Explore, develop, and invent their own algorithms to accomplish a task or to solve numerical problems.

Mathematics Revision 2006
Scope and Sequence by Grade
Grade Three

Strand I – Patterns, Relationships, Functions

Michigan Content Standard:

1. Patterns:

- Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.

Grade Level Benchmark:

EL. 1 Recognize, describe and extend numerical and geometric patterns.

Grade Level Content Expectation:

- N.ME.03.04 Count orally by 6's, 7's, 8's, and 9's starting with 0, making the connection between repeated addition and multiplication.
- N.ME.03.05 Know that even numbers end in 0, 2, 4, 6, or 8; name a whole number quantity that can be shared in two equal groups or grouped into pairs with no remainders; recognize even numbers as multiples of 2. Know that odd numbers end in 1, 3, 5, 7, or 9, and work with patterns involving even and odd numbers.

EL. 2 Represent and record patterns and relationships in a variety of ways including tables, charts and pictures.

EL. 3 Use patterns to describe real-world phenomena.

EL. 4 Explore various types of numeric and geometric patterns (repeating, growing, shrinking).

EL. 5 Apply their experiences with patterns to help solve problems and explore new content.

Michigan Content Standard:

2. Variability and Change:

- Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

Grade Level Benchmark:

EL. 1 Recognize change and variability when it occurs in a variety of settings.

EL. 2 Recognize that change is often predictable, but variable, and that patterns emerge that help to describe the change.

EL. 3 Explore change, and realize that changes are frequently interdependent.

EL. 4 Use tables, charts, open sentences, and hands-on models to represent change and variability.

EL. 5 Begin to describe and differentiate between types of relationships, especially repeating, growing and shrinking patterns.

EL. 6 Explore variability and change in a variety of contexts, investigations and problems.

Strand II: Geometry and Measurement

Michigan Content Standard:

1. Shape and Shape Relationship

- Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

Grade Level Benchmark:

EL. 1 Recognize and name familiar shapes in one, two and three dimensions such as lines, rectangles and spheres and informally discuss the shape of a graph.

Grade Level Content Expectation:

- G.GS.03.01 Identify points, line segments, lines and distance.
- G.GS.03.02 Identify perpendicular lines and parallel lines in familiar shapes and in the classroom.
- G.GS.03.03 Identify parallel faces of rectangular prisms, in familiar shapes and in the classroom. Name, and explore properties of shapes.
- G.GS.03.04 Identify, describe, compare and classify two-dimensional shapes, e.g., parallelogram, trapezoid, circle, rectangle, square and rhombus, based on their component parts (angles, sides, vertices, line segment) and the number of sides and vertices. Explore and name three-dimensional solids.
- G.GS.03.06 Identify, describe, build and classify familiar three-dimensional solids, e.g., cube, rectangular prism, sphere, pyramid, cone, based on their component parts (faces, surfaces, bases, edges, vertices).
- G.SR.03.07 Represent front, top, and side views of solids built with cubes.

EL. 2 Describe the attributes of familiar shapes.

Grade Level Content Expectation:

- G.GS.03.04 Identify, describe, compare and classify two-dimensional shapes, e.g., parallelogram, trapezoid, circle, rectangle, square and rhombus, based on their component parts (angles, sides, vertices, line segment) and the number of sides and vertices.
- G.SR.03.05 Compose and decompose triangles and rectangles to form other familiar two-dimensional shapes; e.g., form a rectangle using two congruent right triangles, or decompose a parallelogram into a rectangle and two right triangles. Explore and name three-dimensional solids.

- G.GS.03.06 Identify, describe, build and classify familiar three-dimensional solids, e.g., cube, rectangular prism, sphere, pyramid, cone, based on their component parts (faces, surfaces, bases, edges, vertices).

EL. 3 Compare, sort and classify familiar shapes.

Grade Level Content Expectation:

- G.GS.03.04 Identify, describe, compare and classify two-dimensional shapes, e.g., parallelogram, trapezoid, circle, rectangle, square and rhombus, based on their component parts (angles, sides, vertices, line segment) and the number of sides and vertices. Explore and name three-dimensional solids.
- G.GS.03.06 Identify, describe, build and classify familiar three-dimensional solids, e.g., cube, rectangular prism, sphere, pyramid, cone, based on their component parts (faces, surfaces, bases, edges, vertices).

EL. 4 Draw and build familiar shapes.

Grade Level Content Expectation:

- G.SR.03.05 Compose and decompose triangles and rectangles to form other familiar two-dimensional shapes; e.g., form a rectangle using two congruent right triangles, or decompose a parallelogram into a rectangle and two right triangles. Explore and name three-dimensional solids.
- G.GS.03.06 Identify, describe, build and classify familiar three-dimensional solids, e.g., cube, rectangular prism, sphere, pyramid, cone, based on their component parts (faces, surfaces, bases, edges, vertices).
- G.SR.03.07 Represent front, top, and side views of solids built with cubes.

EL. 5 Explore ways to combine, dissect, and transform shapes.

Grade Level Content Expectation:

- G.SR.03.05 Compose and decompose triangles and rectangles to form other familiar two-dimensional shapes; e.g., form a rectangle using two congruent right triangles, or decompose a parallelogram into a rectangle and two right triangles. Explore and name three-dimensional solids.
- G.GS.03.06 Identify, describe, build and classify familiar three-dimensional solids, e.g., cube, rectangular prism, sphere, pyramid, cone, based on their component parts (faces, surfaces, bases, edges, vertices).

EL. 6 Recognize parallel and perpendicular line segments and figures that have similarity and/or congruence.

Grade Level Content Expectation:

- G.GS.03.01 Identify points, line segments, lines and distance.
- G.GS.03.02 Identify perpendicular lines and parallel lines in familiar shapes and in the classroom.

EL. 7 Use shape, shape properties and shape relationships to describe the physical world and to solve problems.

Grade Level Content Expectation:

- G.GS.03.02 Identify perpendicular lines and parallel lines in familiar shapes and in the classroom.
- G.GS.03.03 Identify parallel faces of rectangular prisms, in familiar shapes and in the classroom. Name, and explore properties of shapes.
- G.GS.03.04 Identify, describe, compare, and classify two-dimensional shapes, e.g., parallelogram, trapezoid, circle, rectangle, square and rhombus, based on their component parts (angles, sides, vertices, line segment) and the number of sides and vertices.
- G.SR.03.05 Compose and decompose triangles and rectangles to form other familiar two-dimensional shapes; e.g., form a rectangle using two congruent right triangles, or decompose a parallelogram into a rectangle and two right triangles. Explore and name three-dimensional solids.
- G.GS.03.06 Identify, describe, build and classify familiar three-dimensional solids, e.g., cube, rectangular prism, sphere, pyramid, cone, based on their component parts (faces, surfaces, bases, edges, vertices).
- G.SR.03.07 Represent front, top, and side views of solids built with cubes.
- M.PS.03.13 Solve contextual problems about perimeters of rectangles and areas of rectangular regions.

Michigan Content Standard:

2. Position

- Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

Grade Level Benchmark:

EL. 1 Locate and describe objects in terms of their position, including front, back, inside, outside, right, left, over, under, next to, between and locations on the number line on a coordinate graph and on a map.

Grade Level Content Expectation:

- N.ME.03.18 Place fractions with denominators of 2, 4, and 8 on the number line; relate the number line to a ruler; compare and order up to three fractions with denominators 2, 4, and 8.

EL. 2 Locate and describe objects in terms of their orientation, direction and relative position, including up, down, back, N-S-E-W, flipped, turned, translated; recognize symmetrical objects and identify their lines of symmetry.

EL. 3 Explore what happens to the size, shape, and position of an object after sliding, flipping, turning, enlarging or reducing it.

EL. 5 Use concepts of position, direction, and orientation to describe the physical; world and to solve problems.

Michigan Content Standard:

3. Measurement

- Students compare attributes of two objects, or of one with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

Grade Level Benchmark:

EL. 1 Compare attributes of objects; develop standard units of measurement; and select and use standard tools for measurement.

Grade Level Content Expectation:

- M.UN.03.01 Know and use common units of measurements in length, weight and time.
- M.UN.03.02 Measure in mixed units within the same measurement system for length, weight and time: feet and inches, meters and centimeters, kilograms and grams, pounds and ounces, liters and milliliters, hours and minutes, minutes and seconds, years and months.
- M.UN.03.03 Understand relationships between sizes of standard units, e.g., feet and inches, meters and centimeters.
- M.UN.03.04 Know benchmark temperatures such as freezing (32°F, 0°C); boiling (212°F, 100°C); and compare temperatures to these, e.g., cooler, warmer.

EL. 2 Identify the attribute to be measured and select the appropriate unit of measurement for length, mass, weight, area, perimeter, capacity, time, temperature and money.

Grade Level Content Expectation:

- M.UN.03.01 Know and use common units of measurements in length, weight and time. Understand meaning of area and perimeter and apply in problems.
- M.UN.03.05 Know the definition of area and perimeter and calculate the perimeter of a square and rectangle given whole number side lengths.
- M.UN.03.06 Use square units in calculating area by covering the region and counting the number of square units.
- M.UN.03.07 Distinguish between units of length and area and choose a unit appropriate in the context.

EL. 3 Develop strategies for estimating measures and compare the estimates to the results of the measurement; decide if an estimate is “a good estimate.”

Grade Level Content Expectation:

- M.UN.03.08 Visualize and describe the relative sizes of one square inch and one square centimeter. Estimate perimeter and area.
- M.TE.03.09 Estimate the perimeter of a square and rectangle in inches and centimeters; estimate the area of a square and rectangle in square inches and square centimeters.

EL. 4 Explain the meaning of measurements and recognize that the number of units it takes to measure an object is related to the size of the unit.

EL. 5 Explore scale drawings, models, and maps and relate them to measurements of real objects.

EL. 6 Apply measurement to describe the real world and to solve problems.

Grade Level Content Expectation:

- M.PS.03.10 Add and subtract lengths, weights and times using mixed units, within the same measurement system.
- M.PS.03.11 Add and subtract money in dollars and cents.
- M.PS.03.12 Solve applied problems involving money, length and time.
- M.PS.03.13 Solve contextual problems about perimeters of rectangles and areas of rectangular regions.

Strand III: Data Analysis and Statistics

Michigan Content Standard:

1. Collection, Organization, Presentation of Data

- Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.

Grade Level Benchmark:

EL. 1. Collect and explore data through counting, measuring and conducting surveys and experiments.

EL. 2 Organize data using concrete objects, pictures, tallies, tables, charts, diagrams, and graphs.

EL. 3 Present data using a variety of appropriate representations and explain the meaning of the data.

EL. 4 Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize, and present those data.

Michigan Content Standard:

2. Description and Interpretations

- Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.

Grade Level Benchmark:

EL. 1 Read and explain data they have collected and organized themselves and progress to reading data from other sources.

Grade Level Content Expectation:

- D.RE.03.01 Read and interpret bar graphs, in both horizontal and vertical forms.
- D.RE.03.03 Solve problems using information in bar graphs, including comparison of bar graphs.

EL. 2 Describe the shape of the data using informal language.

Grade Level Content Expectation:

- D.RE.03.02 Read scales on the axes and identify the maximum, minimum, and range of values in a bar graph.

EL. 3 Draw, explain, and justify conclusions, such as trends based on data.

Grade Level Content Expectation:

- D.RE.03.03 Solve problems using information in bar graphs, including comparison of bar graphs.

EL. 4 Raise and answer questions about the source, collection, organization and presentation of data, as well as the conclusions drawn from the data; explore biases in data.

EL. 5 Formulate questions and problems and gather and interpret data to answer those questions.

Michigan Content Standard:

3. Inference and Prediction

- Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.

Grade Level Benchmark:

EL. 1 Make and test hypotheses.

EL. 2 Conduct surveys, samplings and experiments to solve problems and answer questions of interest to them.

EL. 3 Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.

EL. 4 Make and explain predictions based on data.

EL. 5 Make predictions to answer questions and solve problems.

Strand IV: Number Sense and Numeration

Michigan Content Standard:

1. Concepts and Properties of Numbers

- Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.

Grade Level Benchmark:

EL. 1 Develop an understanding of whole numbers; and read, write and count using whole numbers; investigate basic concepts of fractions and decimals.

Grade Level Content Expectation:

- N.ME.03.01 Read and write numbers to 10,000 in both numerals and words, and relate them to the quantities they represent, e.g., relate numeral or written word to a display of dots or objects.
- N.ME.03.02 Recognize and use expanded notation for numbers using place value to 10,000s place, e.g., 2,517 is 2 thousands, 5 hundreds, 1 ten, and 7 ones; 4 hundreds and 2 ones is 402; identify the place value of a digit in a number, e.g., in 3,241, 2 is in the hundreds place.
- N.ME.03.03 Compare and order numbers up to 10,000. Count in steps, and understand even and odd numbers.
- N.ME.03.04 Count orally by 6's, 7's, 8's, and 9's starting with 0, making the connection between repeated addition and multiplication. Understand simple fractions, relation to the whole, and addition and subtraction of fractions.
- N.ME.03.16 Understand that fractions may represent a portion of a whole unit that has been partitioned into parts of equal area or length; use the terms "numerator" and "denominator."
- N.ME.03.17 Recognize, name and use equivalent fractions with denominators 2, 4, and 8, using strips as area models.
- N.ME.03.18 Place fractions with denominators of 2, 4, and 8 on the number line; relate the number line to a ruler; compare and order up to three fractions with denominators 2, 4, and 8.
- N.ME.03.20 Recognize that addition and subtraction of fractions with equal denominators can be modeled by joining and taking away segments on the number line.

EL. 2 Investigate and develop an understanding of the base 10 place-value system.

Grade Level Content Expectation:

- N.ME.03.01 Read and write numbers to 10,000 in both numerals and words, and relate them to the quantities they represent, e.g., relate numeral or written word to a display of dots or objects.
- N.ME.03.02 Recognize and use expanded notation for numbers using place value to 10,000s place, e.g., 2,517 is 2 thousands, 5 hundreds, 1 ten, and 7 ones; 4 hundreds and 2 ones is 402; identify the place value of a digit in number, e.g., in 3,241, 2 is in the hundreds place. Understand simple fractions, relation to the whole, and addition and subtraction of fractions.
- N.ME.03.16 Understand that fractions may represent a portion of a whole unit that has been partitioned into parts of equal area or length; use the terms “numerator” and denominator.” Understand simple decimal fractions in relation to money.
- N.ME.03.21 Understand the meaning of \$0.50 and \$0.25 related to money, e.g., \$1.00 shared two people means $\$1.00 \div 2 = 1/2 \text{ dollar} = 0.50$.

EL. 3 Develop an understanding of the properties of numbers (e.g., order) and of the properties of the special numbers 0 and 1.

Grade Level Content Expectation:

- N.ME.03.02 Recognize and use expanded notation for numbers using place value to 10,000s place, e.g., 2,517 is 2 thousands, 5 hundreds, 1 ten, and 7 ones; 4 hundreds and 2 ones is 402; identify the place value of a digit in a number, e.g., in 3,241, 2 is in the hundreds place.

EL. 4 Apply their understanding of number systems to model and solve problems.

Grade Level Content Expectation:

- N.MR.03.14 Solve simple division problems involving remainders, viewing remainder as the “number left over” (less than the divisor), e.g., 4 children per group; we have 25 children; there are 6 groups with 1 child left over; interpret based on problem context. Problem solving with whole numbers.
- N.MR.03.15 Given problems that use any one of the four operations with appropriate numbers, represent with objects, words, (including “product” and “quotient”), and mathematical statements; solve.
- N.ME.03.20 Recognize that addition and subtraction of fractions with equal denominators can be modeled by joining and taking away segments on the number line.

Michigan Content Standard:

2. Representation and Uses of Numbers

- Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Grade Level Benchmark:

EL. 1 Represent whole numbers, fractions, and decimals using concrete, pictorial and symbolic representations.

Grade Level Content Expectation:

- N.ME.03.01 Read and write numbers to 10,000 in both numerals and words, and relate them to the quantities they represent, e.g., relate numeral or written word to a display of dots or objects.
- N.ME.03.02 Recognize and use expanded notation for numbers using place value to 10,000s place, e.g., 2,517 is 2 thousands, 5 hundreds, 1 ten, and 7 ones; 4 hundreds and 2 ones is 402; identify the place value of a digit in a number, e.g., in 3,241, 2 is in the hundreds place. Understand simple fractions, relation to the whole, and addition and subtraction of fractions.
- N.ME.03.16 Understand that fractions may represent a portion of a whole unit that has been partitioned into parts of equal area or length; use the terms “numerator” and “denominator.”
- N.ME.03.17 Recognize, name and use equivalent fractions with denominators 2, 4, and 8, using strips as area models.
- N.ME.03.18 Place fractions with denominators of 2, 4, and 8 on the number line; relate the number line to a ruler; compare and order up to three fractions with denominators 2, 4, and 8. Understand simple decimal fractions in relation to money N.ME.03.21 Understand the meaning of \$0.50 and \$0.25 related to money, e.g., \$1.00 shared by two people means $\$1.00 \div 2 = 1/2 \text{ dollar} = \0.50 .

EL. 2 Explore and recognize different representations for the same number and explain why they are the same.

Grade Level Content Expectation:

- N.ME.03.17 Recognize, name and use equivalent fractions with denominators 2, 4, and 8, using strips as area models.
- N.ME.03.19 Understand that any fraction can be written as a sum of unit fractions, e.g., $3/4 = 1/4 + 1/4 + 1/4$. □

EL. 3 Investigate ways numbers are used (e.g., counting, ordering, naming, locating, measuring).

Grade Level Content Expectation:

- N.ME.03.01 Read and write numbers to 10,000 in both numerals and words, and relate them to the quantities they represent, e.g., relate numeral or written word to a display of dots or objects. Count in steps, and understand even and odd numbers.
- N.ME.03.04 Count orally by 6's, 7's, 8's, and 9's starting with 0, making the connection between repeated addition and multiplication.

EL. 4 Develop strategies for estimating quantity and evaluate the reasonableness of their estimates.

Grade Level Content Expectation:

- N.ME.03.01 Read and write numbers to 10,000 in both numerals and words, and relate them to the quantities they represent, e.g., relate numeral or written word to a display of dots or objects. Count in steps, and understand even and odd numbers.
- N.ME.03.04 Count orally by 6's, 7's, 8's, and 9's starting with 0, making the connection between repeated addition and multiplication.

EL. 5 Select appropriate numbers and representations in order to solve problems.

Grade Level Content Expectation:

- N.MR.03.15 Given problems that use any one of the four operations with appropriate numbers, represent with objects, words, (including “product” and “quotient”), and mathematical statements; solve.

Michigan Content Standard:

3. Number Relationships

- Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Grade Level Benchmark:

EL. 1 Compare and order numbers using “equal,” “less than” or “greater than.”

Grade Level Content Expectation:

- N.ME.03.03 Compare and order numbers up to 10,000. Understand simple fractions, relation to the whole, and addition and subtraction of fractions.
- N.ME.03.17 Recognize, name and use equivalent fractions with denominators 2, 4, and 8, using strips as area models.
- N.ME.03.18 Place fractions with denominators of 2, 4, and 8 on the number line; relate the number line to a ruler; compare and order up to three fractions with denominators 2, 4, and 8.

EL. 2 Use part-whole relationships to explore numbers, develop number concepts, and understand computation.

Grade Level Content Expectation:

- N.ME.03.16 Understand that fractions may represent a portion of a whole unit that has been partitioned into parts of equal area or length; use the terms “numerator” and “denominator.”
- N.ME.03.17 Recognize, name and use equivalent fractions with denominators 2, 4, and 8, using strips as area models.
- N.ME.03.19 Understand that any fraction can be written as a sum of unit fractions, e.g. $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ □.

EL. 3 Classify numbers as even or odd and explore concepts of factors and multiples.

Grade Level Content Expectation:

- N.ME.03.05 Know that even numbers end in 0, 2, 4, 6, or 8; name a whole number quantity that can be shared in two equal groups or grouped into pairs with no remainders; recognize even numbers as multiples of 2. Know that odd numbers end in 1, 3, 5, 7, or 9, and work with patterns involving even and odd numbers.

EL. 5 Apply their understanding of number relationships in solving problems.

Grade Level Content Expectation:

- N.MR.03.15 Given problems that use any one of the four operations with appropriate numbers, represent with objects, words, (including “product” and quotient”), and mathematical statements; solve.

Strand V: Numerical and Algebraic Operations and Analytical Thinking

Michigan Content Standard:

1. Operations and their Properties

- Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.

Grade Level Benchmark:

EL. 1 Use manipulatives to model operations with numbers; develop their own methods of recording operations; and relate their models and recordings to standard symbolic expressions and algorithms.

Grade Level Content Expectation:

- N.MR.03.10 Recognize situations that can be solved using multiplication and division including finding “How many groups?” and “How many in a group?” and write mathematical statements for those situations.
- N.MR.03.14 Solve simple division problems involving remainders, viewing remainder as the “number left over” (less than the divisor), e.g., 4 children per group; we have 25 children; there are 6 groups with 1 child left over; interpret based on problem context.
- N.ME.03.20 Recognize that addition and subtraction of fractions with equal denominators can be modeled by joining and taking away segments on the number line.

EL. 2 Develop and apply the appropriate method of computation from among mental computation, estimation, paper-and-pencil or calculators; explain why they are choosing a method and how they know which operations to perform in a given situation.

Grade Level Content Expectation:

- N.FL.03.06 Add and subtract fluently two numbers: up to and including two-digit numbers with regrouping and up to four digit numbers without regrouping.
- N.FL.03.07 Estimate the sum and difference of two numbers with three digits (sums up to 1000), and judge reasonableness of estimates.
- N.FL.03.08 Use mental strategies to fluently add and subtract two-digit numbers. Multiply and divide whole numbers.
- N.FL.03.11 Find products fluently up to 10×10 ; find related quotients using multiplication and division relationships.
- N.FL.03.13 Mentally calculate simple products and quotients: up to a three-digit number by a one-digit number involving multiples of 10, e.g., 500×6 , or $400 \div 8$.

EL. 3 Explore properties of operations (e.g., commutative and distributive properties) and give examples of how they use those properties.

Grade Level Content Expectation:

- N.ME.03.02 Recognize and use expanded notation for numbers using place value to 10,000s place, e.g., 2,517 is 2 thousands, 5 hundreds, 1 ten, and 7 ones; 4 hundreds and 2 ones is 402; identify the place value of a digit in a number, e.g., in 3,241, 2 is in the hundreds place. Multiply and divide whole numbers.
- N.MR.03.09 Use multiplication and division fact families to understand the inverse relationship of these two operations, e.g., because $3 \times 8 = 24$, we know that $24 \div 8 = 3$ or $24 \div 3 = 8$; express a multiplication statement as an equivalent division statement.

EL. 4 Apply operations efficiently and accurately in solving problems.

Grade Level Content Expectation:

- N.MR.03.15 Given problems that use any one of the four operations with appropriate numbers, represent with objects, words, (including “product” and “quotient”), and mathematical statements; solve.

Michigan Content Standard:

2. Algebraic and Analytic Thinking

- Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Grade Level Benchmark:

EL. 1 Write and solve open sentences (e.g., $x + n = 5$) and write stories to fit the open sentence.

Grade Level Content Expectation:

- N.MR.03.10 Recognize situations that can be solved using multiplication and division including finding “How many groups?” and “How many in a group?” and write mathematical statements for those situations.
- N.MR.03.12 Find solutions to open sentences, such as $7 \times = 42$ or $12 \div = 4$, using the inverse relationship between multiplication and division.

EL. 2 Explore algebraic concepts with manipulatives such as balance scales, tables of input and output, and pictorial representations of problems.

EL. 3 Find replacements for the variable(s) in open sentences.

Grade Level Content Expectation:

- N.MR.03.12 Find solutions to open sentences, such as $7 \times = 42$ or $12 \div = 4$, using the inverse relationship between multiplication and division.

EL. 4 Use analytic thinking to describe situations and solve problems.

Strand VI: Probability and Discrete Mathematics

Michigan Content Standard:

1. Probability

- Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgements about claims that are made in probabilistic situations.

Grade Level Benchmark:

EL. 1 Explain the difference between chance and certainty and give examples to illustrate their understanding.

EL. 2 Compare events and describe them as “more likely” or “less likely” and use the language of fractions to describe simple probabilities.

EL. 3 Conduct experiments with concrete objects to explore concepts and develop an intuitive understanding of how the conditions of the experiment can affect the outcome.

EL. 4 Conduct experiments, record the outcomes, and examine those outcomes to determine if they make sense and search for explanations of the outcomes.

EL. 5 Conduct probability experiments and simulations to model and solve problems.

Michigan Content Standard:

2. Discrete Mathematics

- Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.

Grade Level Benchmark:

EL. 1 Use manipulatives and diagrams to explore problems involving counting and arranging objects.

Grade Level Content Expectation:

- N.ME.03.20 Recognize that addition and subtraction of fractions with equal denominators can be modeled by joining and taking away segments on the number line.

EL. 2 Explore sets and set relationships by sorting and classifying objects.

EL. 3 Explore situations in which they model and trace paths using figures consisting of vertices connected by edges.

EL. 4 Explore now-next patterns.

EL. 5 Explore, develop, and invent their own algorithms to accomplish a task or to solve numerical problems.

EL. 6 Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems) and decide upon a best solution (optimization problems).

Mathematics Revision 2006
Scope and Sequence by Grade
Grade Four

Strand I – Patterns, Relationships, Functions

Michigan Content Standard:

1. Patterns:

- Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.

Grade Level Benchmark:

EL. 1. Recognize, describe, and extend numerical and geometric patterns.

EL. 2. Represent and record patterns and relationships in a variety of ways including tables, charts and pictures.

EL. 3. Use patterns to describe real-world phenomena.

EL. 4. Explore various types of numeric and geometric patterns (repeating, growing, shrinking).

EL. 5. Apply their experiences with patterns to help solve problems and explore new content.

Michigan Content Standard:

2. Variability and Change:

- Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

Grade Level Benchmark:

EL. 1. Recognize change and variability when it occurs in a variety of settings.

EL. 2. Recognize that change is often predictable, but variable, and that patterns emerge that help to describe the change.

EL. 3. Explore change, and realize that changes are frequently interdependent.

EL. 4. Use tables, charts, open sentences, and hands-on models to represent change and variability.

EL. 5. Begin to describe and differentiate between types of relationships, especially repeating, growing, and shrinking patterns.

EL. 6. Explore variability and change in a variety of contexts, investigations and problems.

Strand II: Geometry and Measurement

Michigan Content Standard:

1. Shape and Shape Relationship

- Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

Grade Level Benchmark:

EL. 1. Recognize and name familiar shapes in one, two and three dimensions such as lines, rectangles and spheres and informally discuss the shape of a graph:

Grade Level Content Expectation:

- G.GS.04.01 Identify and draw perpendicular, parallel, and intersecting lines using a ruler and a tool or object with a square (90°) corner.
- G.GS.04.02 Identify basic geometric shapes, including isosceles, equilateral and right triangles, and use their properties to solve problems.
- G.SR.04.03 Identify and count the faces, edges, and vertices of basic three-dimensional geometric solids including cubes, rectangular prisms, and pyramids; describe the shape of their faces.
- M.TE.04.10 Identify right angles and compare angles to right angles.

EL.2. Describe the attributes of familiar shapes.

Grade Level Content Expectation:

- G.GS.04.02 Identify basic geometric shapes, including isosceles, equilateral and right triangles, and use their properties to solve problems.
- G.SR.04.03 Identify and count the faces, edges, and vertices of basic three dimensional geometric solids including cubes, rectangular prisms, and pyramids; describe the shape of their faces.
- M.TE.04.10 Identify right angles and compare angles to right angles.

EL. 3. Compare, sort, and classify familiar shapes.

Grade Level Content Expectation:

- G.GS.04.02 Identify basic geometric shapes, including isosceles, equilateral and right triangles, and use their properties to solve problems.
- G.SR.04.03 Identify and count the faces, edges, and vertices of basic three dimensional geometric solids including cubes, rectangular prisms, and pyramids; describe the shape of their faces.

EL. 4. Draw and build familiar shapes: classify familiar shapes.

Grade Level Content Expectation:

- G.GS.04.01 Identify and draw perpendicular, parallel, and intersecting lines using a ruler and a tool or object with a square (90°) corner.

EL. 5. Explore ways to combine, dissect, and transform shapes.

Grade Level Content Expectation:

- G.TR.04.04 Recognize plane figures that have line symmetry.
- G.TR.04.05 Recognize rigid motion transformations (flips, slides, turns) of a two-dimensional object.

EL. 6. Recognize parallel and perpendicular line segments and figures that have similarity and/or congruence.

Grade Level Content Expectation:

- G.GS.04.01 Identify and draw perpendicular, parallel, and intersecting lines using a ruler and a tool or object with a square (90°) corner.
- G.TR.04.05 Recognize rigid motion transformations (flips, slides, turns) of a two-dimensional object.

EL. 7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.

Grade Level Content Expectation:

- M.TE.04.06 Know and understand the formulas for perimeter and area of a square and a rectangle; calculate the perimeters and areas of these shapes and combinations of these shapes using the formulas.
- M.TE.04.07 Find one dimension of a rectangle given the other dimension and its perimeter or area.
- M.TE.04.08 Find the side of a square given its perimeter or area.
- M.PS.04.09 Solve contextual problems about perimeter and area of squares and rectangles in compound shapes.
- M.PS.04.11 Solve contextual problems about surface area.

Michigan Content Standard:

2. Position

- Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

Grade Level Benchmark:

EL. 1. Locate and describe objects in terms of their position, including front, back, inside, outside, right, left, over, under, next to, between and locations on the number line on a coordinate graph and on a map.

EL. 2. Locate and describe objects in terms of their orientation, direction and relative position, including up, down, back, N-S-E-W, flipped, turned, translated; recognize symmetrical objects and identify their lines of symmetry.

Grade Level Content Expectation:

- G.TR.04.04 Recognize plane figures that have line symmetry.
- G.TR.04.05 Recognize rigid motion transformations (flips, slides, turns) of a two-dimensional object.

EL. 3. Explore what happens to the size, shape, and position of an object after sliding, flipping, turning, enlarging or reducing it.

Grade Level Content Expectation:

- G.TR.04.04 Recognize plane figures that have line symmetry.
- G.TR.04.05 Recognize rigid motion transformations (flips, slides, turns) of a two-dimensional object.

EL. 5. Use concepts of position, direction, and orientation to describe the physical; world and to solve problems.

Grade Level Content Expectation:

- G.TR.04.04 Recognize plane figures that have line symmetry.
- G.TR.04.05 Recognize rigid motion transformations (flips, slides, turns) of a two-dimensional object.

Michigan Content Standard:

3. Measurement

- Students compare attributes of two objects, or of one with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

Grade Level Benchmark:

EL. 1. Compare attributes of objects; develop standard units of measurement; and select and use standard tools for measurement.

Grade Level Content Expectation:

- G.GS.04.01 Identify and draw perpendicular, parallel, and intersecting lines using a ruler and a tool or object with a square (90°) corner.
- M.UN.04.01 Measure using common tools and select appropriate units of measure.
- M.UN.04.03 Measure and compare integer temperatures in degrees.

EL. 2. Identify the attribute to be measured and select the appropriate unit of measurement for length, mass, weight, area, perimeter, capacity, time, temperature and money.

Grade Level Content Expectation:

- M.UN.04.01 Measure using common tools and select appropriate units of measure.

EL. 3. Develop strategies for estimating measures and compare the estimates to the results of the measurement; decide if an estimate is “a good estimate.”

Grade Level Content Expectation:

- M.PS.04.02 Give answers to a reasonable degree of precision in the context of a given problem.

EL. 4. Explain the meaning of measurements and recognize that the number of units it takes to measure an object is related to the size of the unit.

Grade Level Content Expectation:

- M.TE.04.04 Measure surface area of cubes and rectangular prisms by covering and counting area of the faces.
- M.TE.04.05 Carry out the following conversions from one unit of measure to a larger or smaller unit of measure: meters to centimeters, kilograms to grams, liters to milliliters, hours to minutes, minutes to seconds, years to months, weeks to days, feet to inches, ounces to pounds (using numbers that involve only simple calculations.)

EL. 5. Explore scale drawings, models, and maps and relate them to measurements of real objects.

EL. 6. Apply measurement to describe the real world and to solve problems.

Grade Level Content Expectation:

- M.PS.04.09 Solve contextual problems about perimeter and area of squares and rectangles in compound shapes.
- M.PS.04.11 Solve contextual problems about surface area.

Strand III: Data Analysis and Statistics

Michigan Content Standard:

1. Collection, Organization, Presentation of Data

- Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.

Grade Level Benchmark:

EL. 1. Collect and explore data through counting, measuring, and conducting surveys and experiments.

Grade Level Content Expectation:

- D.RE.04.01 Construct tables and bar graphs from given data.
- D.RE.04.02 Order a given set of data, find the median, and specify the range of values.

EL. 2. Organize data using concrete objects, pictures, tallies, tables, charts, diagrams and graphs.

Grade Level Content Expectation:

- D.RE.04.01 Construct tables and bar graphs from given data.
- D.RE.04.03 Solve problems using data presented in tables and bar graphs, e.g., compare data represented in two bar graphs; read bar graphs showing two data sets.

EL. 3. Present data using a variety of appropriate representations and explain the meaning of the data.

Grade Level Content Expectation:

- D.RE.04.03 Solve problems using data presented in tables and bar graphs, e.g., compare data represented in two bar graphs; read bar graphs showing two data sets.

EL. 4. Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize, and present those data.

Michigan Content Standard:

2. Description and Interpretations

- Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.

Grade Level Benchmark:

EL. 1. Read and explain data they have collected and organized themselves and progress to reading data from other sources.

Grade Level Content Expectation:

- D.RE.04.03 Solve problems using data presented in tables and bar graphs, e.g., compare data represented in two bar graphs; read bar graphs showing two data sets.

EL. 2. Describe the shape of the data using informal language.

Grade Level Content Expectation:

- D.RE.04.02 Order a given set of data, find the median, and specify the range of values.

EL. 3. Draw, explain, and justify conclusions, such as trends based on data.

Grade Level Content Expectation:

- D.RE.04.03 Solve problems using data presented in tables and bar graphs, e.g., compare data represented in two bar graphs; read bar graphs showing two data sets.

EL. 4. Raise and answer questions about the source, collection, organization and presentation of data, as well as the conclusions drawn from the data; explore biases in data.

Grade Level Content Expectation:

- D.RE.04.03 Solve problems using data presented in tables and bar graphs, e.g., compare data represented in two bar graphs; read bar graphs showing two data sets.

EL. 5. Formulate questions and problems and gather and interpret data to answer those questions.

Grade Level Content Expectation:

- D.RE.04.01 Construct tables and bar graphs from given data.

Michigan Content Standard:

3. Inference and Prediction

- Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.

Grade Level Benchmark:

EL. 1. Making and testing hypotheses is an essential technique for gaining new knowledge.

EL. 2. Conduct surveys, samplings, and experiments to solve problems and answer questions of interest to them.

EL. 3. Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.

Grade Level Content Expectation:

- D.RE.04.03 Solve problems using data presented in tables and bar graphs, e.g., compare data represented in two bar graphs; read bar graphs showing two data sets.

EL. 4. Make and explain predictions based on data.

EL. 5. Make predictions to answer questions and solve problems.

Strand IV: Number Sense and Numeration

Michigan Content Standard:

1. Concepts and Properties of Numbers

- Number sense is to mathematics what vocabulary is to language. Students must learn to quantify and measure, concretely at first and increasingly more abstractly as they mature. They also must develop an understanding of numeration systems and of the structure of such systems. They must learn to estimate mathematical quantities and to represent and communicate mathematical ideas in the language of mathematics.

Grade Level Benchmark:

EL. 1. Develop an understanding of whole numbers; and read, write and count using whole numbers; investigate basic concepts of fractions and decimals.

Grade Level Content Expectation:

- N.ME.04.01 Read and write numbers to 1,000,000; relate them to the quantities they represent; compare and order.
- N.ME.04.02 Compose and decompose numbers using place value to 1,000,000's, e.g., 25,068 is 2 ten thousands, 5 thousands, 0 hundreds, 6 tens, and 8 ones.
- N.ME.04.03 Understand the magnitude of numbers up to 1,000,000; recognize the place value's of numbers, and the relationship of each place value to the place to its right, e.g., 1,000 is 10 hundreds. Read, interpret and compare decimal fractions.
- N.ME.04.15 Read and interpret decimals up to two decimal places; relate to money and place value decomposition.
- N.ME.04.16 Know that terminating decimals represent fractions whose denominators are 10, 10 x 10, 10 x 10 x 10, etc. e.g., powers of 10.
- N.ME.04.17 Locate tenths and hundredths on a number line.
- N.ME.04.18 Read, write, interpret, and compare decimals up to two decimal places.
- N.ME.04.20 Understand fractions as parts of a set of objects.

EL. 2. Investigate and develop an understanding of the base 10, place-value system.

Grade Level Content Expectation:

- N.ME.04.15 Read and interpret decimals up to two decimal places; relate to money and place value decomposition.
- N.ME.04.17 Locate tenths and hundredths on a number line.
- N.ME.04.18 Read, write, interpret, and compare decimals up to two decimal places.

- N.MR.04.19 Write tenths and hundredths in decimal and fraction forms, and know the decimal equivalents for halves and fourths.

EL. 3. Develop an understanding of the properties of numbers (e.g., order) and of the properties of the special numbers 0 and 1.

Grade Level Content Expectation:

- N.ME.04.02 Compose and decompose numbers using place value to 1,000,000's, e.g., 25,068 is 2 ten thousands, 5 thousands, 0 hundreds, 6 tens, and 8 ones.
- N.ME.04.03 Understand the magnitude numbers up to 1,000,000; recognize the place value's of numbers, and the relationship of each place value to the place to its right, e.g., 1,000 is 10 hundreds.
- N.ME.04.09 Multiply two-digit numbers by 2, 3, 4, and 5, using the distributive property, e.g., $21 \times 3 = (1 + 20) \times 3 = (13) + (20 \times 3) = 3 + 60 = 63$'s.

EL. 4. Apply their understanding of number systems to model and solve problems.

Grade Level Content Expectation:

- N.MR.04.31 Use mathematical statements to represent problems that use addition and subtraction of decimals with up to two-digits; solve.
- N.MR.04.37 Solve applied problems using the four basic arithmetic operations, for appropriate fractions, decimals, and whole numbers.

Michigan Content Standard:

2. Representation and Uses of Numbers

- Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Grade Level Benchmark:

EL. 1. Represent whole numbers, fractions, and decimals using concrete, pictorial and symbolic representations.

Grade Level Content Expectation:

- N.ME.04.03 Understand the magnitude of numbers up to 1,000,000; recognize the place value's of numbers, and the relationship of each place value to the place to its right, e.g., 1,000 is 10 hundreds.
- N.ME.04.15 Read and interpret decimals up to two decimal places; relate to money and place value decomposition.
- N.ME.04.16 Know that terminating decimals represent fractions whose denominators are 10, 10×10 , $10 \times 10 \times 10$, etc. e.g., powers of 10.
- N.ME.04.18 Read, write, interpret, and compare decimals up to two decimal places.
- N.MR.04.19 Write tenths and hundredths in decimal and fraction forms, and know the decimal equivalents for halves and fourths.
- N.ME.04.20 Understand fractions as parts of a set of objects.
- N.MR.04.22 Locate and compare fractions on the number line, including improper fractions and mixed numbers with denominators of 12 or less.
- N.MR.04.24 Know that fractions of the form m/n where m is greater than n , are greater than 1 and are called improper fractions; locate improper fractions on the number line; express as mixed numbers.
- N.MR.04.25 Write improper fractions as mixed numbers, and understand that a mixed number represents the number of "wholes" and the part of a whole remaining, e.g., $5/4 = 1 + 1/4$.

EL. 2. Explore and recognize different representations for the same number and explain why they are the same.

Grade Level Content Expectation:

- N.ME.04.18 Read, write, interpret, and compare decimals up to two decimal places.
- N.MR.04.21 Explain why equivalent fractions are equal, using models such as fraction strips or the number line, for fractions with denominators of 12 or less, or equal to 100.
- N.MR.04.22 Locate and compare fractions on the number line, including improper fractions and mixed numbers with denominators of 12 or less.
- N.MR.04.23 Understand the relationships among halves, fourths and eighths and among thirds, sixths and twelfths.

- N.MR.04.24 Know that fractions of the form $\frac{n}{m}$, where m is greater than n , are greater than 1 and are called improper fractions; locate improper fractions on the number line; express as mixed numbers.
- N.MR.04.26 Compare and order up to three fractions with denominators 2, 4, and 8, and 3, 6, and 12, including improper fractions and mixed numbers.

EL. 3. Investigate ways numbers are used (e.g., counting, ordering, naming, locating, measuring).

Grade Level Content Expectation:

- N.ME.04.01 Read and write numbers to 1,000,000; relate them to the quantities they represent; compare and order.

EL. 4. Develop strategies for estimating quantity and evaluate the reasonableness of their estimates.

Grade Level Content Expectation:

- N.FL.04.34 Estimate the answers to calculations involving addition, subtraction, or multiplication.
- N.FL.04.35 Know when approximation is appropriate and use it to check the reasonableness of answers; be familiar with common place-value errors in calculations.
- N.FL.04.36 Make appropriate estimations and calculations fluently with whole numbers using mental math strategies.

EL. 5. Select appropriate numbers and representations in order to solve problems.

Grade Level Content Expectation:

- N.MR.04.30 Multiply fractions by whole numbers, using repeated addition and area or array models.
- N.MR.04.37 Solve applied problems using the four basic arithmetic operations, for appropriate fractions, decimals, and whole numbers.

Michigan Content Standard:

3. Number Relationships

- Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Grade Level Benchmark:

EL. 1. Compare and order numbers using “equal,” “less than” or “greater than.”

Grade Level Content Expectation:

- N.ME.04.03 Understand the magnitude of numbers up to 1,000,000; recognize the place value’s of numbers, and the relationship of each place value to the place to its right, e.g., 1,000 is 10 hundreds. Read, interpret and compare decimal fractions.
- N.ME.04.17 Locate tenths and hundredths on a number line.
- N.ME.04.18 Read, write, interpret, and compare decimals up to two decimal places.
- N.MR.04.23 Understand the relationships among halves, fourths and eighths and among thirds, sixths and twelfths.
- N.MR.04.26 Compare and order up to three fractions with denominators 2, 4, and 8, and 3, 6, and 12, including improper fractions and mixed numbers.

EL. 2. Use part-whole relationships to explore numbers, develop number concepts and understand computation.

Grade Level Content Expectation:

- N.ME.04.04 Find all factors of a whole number up to 50, and list factor pairs.
- N.ME.04.20 Understand fractions as parts of a set of objects.
- N.MR.04.21 Explain why equivalent fractions are equal, using models such as fraction strips or the number line, for fractions with denominators of 12 or less, or equal to 100.
- N.MR.04.22 Locate and compare fractions on the number line, including improper fractions and mixed numbers with denominators of 12 or less.

EL. 3. Classify numbers as even or odd and explore concepts of factors and multiples.

Grade Level Content Expectation:

- N.ME.04.04 Find all factors of a whole number up to 50, and list factor pairs.
- N.ME.04.05 List the first ten multiples of a given one-digit whole number; determine if a whole number is a multiple of a given one-digit whole number, and if a one-digit number is a factor of a given whole number.
- N.MR.04.06 Know that some numbers, including 2, 3, 5, 7, and 11 have exactly two factors (1 and the number itself) and are called prime numbers.

- N.MR.04.07 Solve problems about factors and multiples, e.g., since $100 = 4 \times 25$, and $200 = 2 \times 100$, then $200 = 2 \times 4 \times 25 = 8 \times 25$.

EL. 5. Apply their understanding of number relationships in solving problem

Grade Level Content Expectation:

- N.MR.04.37 Solve applied problems using the four basic arithmetic operations, for appropriate fractions, decimals, and whole numbers.

Strand V: Numerical and Algebraic Operations and Analytical Thinking

Michigan Content Standard:

1. Operations and their Properties

- Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.

Grade Level Benchmark:

EL. 1. Use manipulatives to model operations with numbers; develop their own methods of recording operations; and relate their models and recordings to standard symbolic expressions and algorithms.

Grade Level Content Expectation:

- N.MR.04.30 Multiply fractions by whole numbers, using repeated addition and area or array models.

EL. 2. Develop and apply the appropriate method of computation from among mental computation, estimation, paper-and-pencil or calculators; explain why they are choosing a method and how they know which operations to perform in a given situation.

Grade Level Content Expectation:

- N.FL.04.08 Add and subtract whole numbers fluently.
- N.ME.04.09 Multiply two-digit numbers by 2, 3, 4, and 5, using the distributive property, e.g., $21 \times 3 = (1 + 20) \times 3 = (1 \times 3) + (20 \times 3) = 3 + 60 = 63$.
- N.FL.04.10 Multiply fluently any whole number by a one-digit number, and a three -digit number by a two-digit number; for two-digit by one-digit multiplication, use distributive property to develop meaning for the algorithm.
- N.FL.04.11 Divide numbers up to four digits by one-digit numbers and by 10.
- N.MR.04.27 Add and subtract fractions less than 1 with denominators 12 or less and including 100, in cases where the denominators are equal or when one denominator is a multiple of the other; e.g., $1/12 + 5/12 = 6/12$; $2/25 + 7/50 = 11/50$.
- N.FL.04.28 Solve fraction problems involving sums and differences for fractions where one denominator is a multiple of the other (denominators 2 through 12, and 100).
- N.FL.04.32 Add and subtract decimals up to two decimal places.
- N.FL.04.33 Multiply and divide decimals up to two decimal places by a one-digit whole number where the result is a terminating decimal, e.g., $0.42 \div 3 = 0.14$, but not $5 \div 3 = 1$.
- N.FL.04.34 Estimate the answers to calculations involving addition, subtraction, or multiplication.

- N.FL.04.35 Know when approximation is appropriate and use it to check the reasonableness of answers; be familiar with common place-value errors in calculations.
- N.FL.04.36 Make appropriate estimations and calculations fluently with whole numbers using mental math strategies.

EL. 3. Explore properties of operations (e.g., commutative and distributive properties) and give examples of how they use those properties.

Grade Level Content Expectation:

- N.ME.04.02 Compose and decompose numbers using place value to 1,000,000's, e.g., 25,068 is 2 ten thousands, 5 thousands, 0 hundreds, 6 tens, and 8 ones.
- N.ME.04.09 Multiply two-digit numbers by 2, 3, 4, and 5, using the distributive property, e.g., $21 \times 3 = (1 + 20) \times 3 = (1 \times 3) + (20 \times 3) = 3 + 60 = 63$.
- N.FL.04.10 Multiply fluently any whole number by a one-digit number, and a three-digit number by a two-digit number; for two-digit by one-digit multiplication, use distributive property to develop meaning for the algorithm.
- N.MR.04.13 Use the relationship between multiplication and division to simplify computations and check results.
- N.ME.04.16 Know that terminating decimals represent fractions whose denominators are 10, 10×10 , $10 \times 10 \times 10$, etc. e.g., powers of 10.

EL. 4. Apply operations efficiently and accurately in solving problems.

Grade Level Content Expectation:

- N.FL.04.14 Solve applied problems involving whole number multiplication and division.
- N.FL.04.28 Solve fraction problems involving sums and differences for fractions where one denominator is a multiple of the other (denominators 2 through 12, and 100).
- N.MR.04.31 Use mathematical statements to represent problems that use addition and subtraction of decimals with up to two-digits; solve.
- N.MR.04.37 Solve applied problems using the four basic arithmetic operations, for appropriate fractions, decimals, and whole numbers.

Michigan Content Standard:

2. Algebraic and Analytic Thinking

- Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Grade Level Benchmark:

EL. 1. Write and solve open sentences (e.g., # + @ = 5) and write stories to fit the open sentence.

Grade Level Content Expectation:

- N.FL.04.12 Find unknowns in equations such as $a \div 10 = 25$; $125 \div b = 25$.

EL. 2. Explore algebraic concepts with manipulatives such as balance scales, tables of input and output, and pictorial representations of problems.

EL. 3. Find replacements for the variable(s) in open sentences.

Grade Level Content Expectation:

- N.FL.04.12 Find unknowns in equations such as $a/10 = 25$; $125/b = 25$.
- N.MR.04.29 Solve for the unknown in equations such as: $1/8 + x = 5/8$ or $3/4 - y = 1/2$.

EL. 4. Use analytic thinking to describe situations and solve problems.

Strand VI: Probability and Discrete Mathematics

Michigan Content Standard:

1. Probability

- Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations.

EL. 1. Explain the difference between chance and certainty and give examples to illustrate their understanding.

EL. 2. Compare events and describe them as “more likely” or “less likely” and use the language of fractions to describe simple probabilities.

EL. 3. Conduct experiments with concrete objects to explore concepts and develop an intuitive understanding of how the conditions of the experiment can affect the outcome.

EL. 4. Conduct experiments, record the outcomes, and examine those outcomes to determine if they make sense and search for explanations of the outcomes.

EL. 5. Conduct probability experiments and simulations to model and solve problems.

Michigan Content Standard:

2. Discrete Mathematics

- Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.

Grade Level Benchmark:

EL. 1. Use manipulatives and diagrams to explore problems involving counting and arranging objects.

EL. 2. Explore sets and set relationships by sorting and classifying objects.

EL. 3. Explore situations in which they model and trace paths using figures consisting of vertices connected by edges.

EL. 4. Explore now-next patterns.

EL. 5. Explore, develop and invent their own algorithms to accomplish a task or to solve numerical problems.

EL. 6. Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems) and decide upon a best solution (optimization problems).

Mathematics
Scope and Sequence
Grade Five

Strand I – Patterns, Relationships, Functions

Michigan Content Standard:

1. Patterns:

- Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.

Grade Level Benchmark:

MS. 1. Recognize, describe and extend numerical and geometric patterns.

MS. 2. Represent and record patterns in a variety of ways including tables, charts and graphs, and translate between various representations.

MS. 3. Use patterns and their generalizations to make and justify inferences and predictions.

MS. 4. Explore and describe visual and numeric patterns, including linear expressions, near-linear patterns and symmetric and spatial patterns.

MS. 5. Use patterns and generalizations to solve problems and explore new content.

Michigan Content Standard:

2. Variability and Change:

- Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

Grade Level Benchmark:

MS. 1. Identify and describe the nature of change; recognize change in more abstract and complex situations and explore different kinds of change, and patterns of variation.

Grade Level Content Expectation:

- D.RE.05.01 Read and interpret line graphs, and solve problems based on line graphs, e.g., distance - time graphs, and problems with two or three line graphs on same axes, comparing different data.

MS. 2. Connect an initial state to a final state and generalize a rule that describes a pattern of change.

MS. 3. Begin to investigate applications in bivariate data and linear relationships, and explore questions of what will happen to one quantity if another variable is changed.

MS. 4. Represent variability of change by ordered pairs, tables, graphs, and equations.

MS. 5. Differentiate between functions and relationships such as linear vs. not linear or continuous vs. non-continuous.

MS. 6. Continue to explore relationships arising from interesting contexts, and use variables and relationships to solve mathematical problems.

Strand II: Geometry and Measurement

Michigan Content Standard:

1. Shape and Shape Relationship

- Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

Grade Level Benchmark:

MS. 1. Distinguish among shapes and differentiate between examples and non-examples of shapes based on their properties; generalize about shapes of graphs and data distributions.

MS. 2. Generalize the characteristics of shapes and apply their generalizations to classes of shapes.

Grade Level Content Expectation:

- G.GS.05.02 Measure angles with a protractor and classify them as acute, right, obtuse or straight.
- G.GS.05.06 Understand why the sum of the interior angles of a triangle is 180° and the sum of the interior angles of a quadrilateral is 360° , and use these properties to solve problems.

MS. 3. Derive generalizations about shapes and apply those generalizations to develop classifications of familiar shapes.

MS. 4. Construct familiar shapes using coordinates, appropriate tools (including technology), sketching and drawing two- and three-dimensional shapes.

Grade Level Content Expectation:

- M.TE.05.08 Build solids with unit cubes and state their volumes.

MS. 5. Combine, dissect, and transform shapes.

MS. 6. Generalize about the common properties of similar, congruent, parallel and perpendicular shapes and verify their generalizations informally.

MS. 7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.

Michigan Content Standard:

2. Position

- Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

Grade Level Benchmark:

MS. 1. Locate and describe objects in terms of their position, including compass directions, Cartesian coordinates, latitude and longitude, and midpoints.

MS. 2. Locate and describe objects in terms of their orientation and relative position, including coincident, collinear, parallel, perpendicular; differentiate between fixed (e.g., N-S-E-W) and relative (e.g., right-left) orientations; recognize and describe Examples of bilateral and rotational symmetry.

MS. 3. Describe translations, reflections, rotations, and dilations using the language of transformations, and employ transformations to verify congruence of figures.

Grade Level Content Expectation:

- G.TR.05.01 Associate an angle with a certain amount of turning; know that angles are measured in degrees; understand that 90° , 180° , 270° , and 360° are associated, respectively, with $1/4$, $1/2$, $4/3$ and full turns.

MS. 4. Locate the position of points or objects described by two or more conditions; locate all the points (locus) that satisfy a given condition.

MS. 5. Use concepts of position, direction, and orientation to describe the physical world and to solve problems.

Michigan Content Standard:

3. Measurement

- Students compare attributes of two objects, or of one with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

Grade Level Benchmark:

MS. 1. Select and use appropriate tools; measure objects using standard units in both the metric and common systems, and measure angles in degrees.

Grade Level Content Expectation:

- M.TE.05.09 Use filling (unit cubes or liquid), and counting or measuring to find the volume of a cube and rectangular prism.
- G.TR.05.01 Associate an angle with a certain amount of turning; know that angles are measured in degrees; understand that 90° , 180° , 270° , and 360° are associated, respectively, with $1/4$, $1/2$, $4/3$ and full turns.
- G.GS.05.02 Measure angles with a protractor and classify them as acute, right, obtuse or straight.

MS. 2. Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), time, temperature, perimeter, area, volume, angle.

Grade Level Content Expectation:

- M.UN.05.01 Recognize the equivalence of 1 liter, 1000 ml and 1000 cm³ and include conversions among liters, milliliters, and cubic centimeters.
- M.UN.05.02 Know the units of measure of volume: cubic centimeter, cubic meter, cubic inches, cubic feet, cubic yards, and use their abbreviations: cm³, m³, in³, ft³, yd³.
- M.UN.05.03 Compare the relative sizes of one cubic inch to one cubic foot, and one cubic centimeter to one cubic meter.

MS. 3. Estimate measures with a specified degree of accuracy and decide if an estimate or a measurement is “close enough”.

MS. 4. Interpret measurements and recognize that two objects may have the same measurement on one attribute (e.g., area), but not necessarily on another (e.g., perimeter).

Grade Level Content Expectation:

M.PS.05.05 Represent relationships between areas of rectangles, triangles and parallelograms using models.

MS. 5. Use proportional reasoning and indirect measurements to draw inferences.

Grade Level Content Expectation:

- M.UN.05.01 Recognize the equivalence of 1 liter, 1000 ml and 1000 cm³ and include conversions among liters, milliliters, and cubic centimeters.
- M.UN.05.04 Convert measurements of length, weight, area, volume, and time within a given system, using easily manipulated numbers.
- M.TE.05.06 Understand and know how to use the area formula of a triangle: $A = \frac{1}{2}bh$ (where b is length of the base and h is the height), and represent using models and manipulatives.
- M.TE.05.07 Understand and know how to use the area formula for a parallelogram: $A = bh$, and represent using models and manipulatives.
- G.GS.05.03 Identify and name angles on a straight line and vertical angles.
- G.GS.05.04 Find unknown angles in problems involving angles on a straight line, angles surrounding point, and vertical angles.
- G.GS.05.05 Know that angles on a straight line add up to 180° and angles surrounding a point add up to 360°; justify informally by “surrounding” a point with angles.

- G.GS.05.07 Find unknown angles using the properties of: triangles, including right, isosceles, and equilateral triangles; parallelograms, including rectangles and rhombuses; and trapezoids.

MS. 6. Apply measurement to describe the real world and to solve problems.

Grade Level Content Expectation:

- M.PS.05.10 Solve applied problems about the volumes of rectangular prisms using multiplication and division and using the appropriate units.

Strand III: Data Analysis and Statistics

Michigan Content Standard:

1. Collection, Organization, Presentation of Data

- Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.

Grade Level Benchmark:

MS. 1. Collect and explore data through observation, measurement, surveys, sampling techniques, and simulations.

MS. 2. Organize data using tables, charts, graphs, spreadsheets and databases.

Grade Level Content Expectation:

- D.RE.05.02 Construct line graphs from tables of data; include axis labels and scale.

MS. 3. Present data using a variety of appropriate representations and explain why one representation is preferred over another or how a particular representation may bias the presentation.

Grade Level Content Expectation:

- D.RE.05.01 Read and interpret line graphs, and solve problems based on line graphs, e.g., distance - time graphs, and problems with two or three line graphs on same axes, comparing different data.
- D.RE.05.02 Construct line graphs from tables of data; include axis labels and scale.

MS. 4. Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.

Michigan Content Standard:

2. Description and Interpretations

- Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.

Grade Level Benchmark:

MS. 1. Critically read data from tables, charts or graphs and explain the source of the data and what the data represent.

MS. 2. Describe the shape of a data distribution and identify the center, the spread, correlations and any outliers.

Grade Level Content Expectation:

- D.AN.05.03 Given a set of data, find and interpret the mean (using the concept of fair share) and mode.

MS. 3. Draw, explain, and justify conclusions based on data.

Grade Level Content Expectation:

- D.RE.05.01 Read and interpret line graphs, and solve problems based on line graphs, e.g., distance time graphs, and problems with two or three line graphs on same axes, comparing different data.
- D.AN.05.04 Solve multi-step problems involving means.

MS. 4. Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the possible sources of bias in the data or their presentation.

Grade Level Content Expectation:

- D.RE.05.01 Read and interpret line graphs, and solve problems based on line graphs, e.g., distance - time graphs, and problems with two or three line graphs on same axes, comparing different data.

MS. 5. Formulate questions and problems and gather and interpret data to answer those questions.

Michigan Content Standard:

3. Inference and Prediction

- Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.

Grade Level Benchmark:

MS. 1. Make and test hypotheses. Formulate questions and problems and gather and interpret data to answer those questions.

MS. 2. Design experiments to model and solve problems using sampling, simulations and controlled investigations.

MS. 3. Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.

MS. 4. Make predictions and decisions based on data, including interpolations and extrapolations.

MS. 5. Employ investigations, mathematical models and simulations to make inferences and predictions to answer questions and solve problems.

Strand IV: Number Sense and Numeration

Michigan Content Standard:

1. Concepts and Properties of Numbers

- Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.

Grade Level Benchmark:

MS. 1. Develop an understanding of integers and rational numbers and represent rational numbers in both fraction and decimal form.

Grade Level Content Expectation:

- N.ME.05.10 Understand a fraction as a statement of division, e.g., $2 \div 3 = \frac{2}{3}$, using simple fractions and pictures to represent.

MS. 2. Extend their understanding of numeration systems to include decimal numeration, scientific numeration and non-decimal numeration systems.

MS. 3. Develop an understanding of the properties of the integer and rational number systems (e.g., order, density) and of the properties of special numbers including 0, 1 and $\frac{1}{2}$, and the additive and multiplicative inverses.

MS. 4. Apply their understanding of number systems to model and solve mathematical and applied problems.

Michigan Content Standard:

2. Representation and Uses of Numbers

- Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Grade Level Benchmark:

MS. 1. Give geometric representations of fractions, prime and composite numbers, triangular and square numbers, and other number concepts; represent rational numbers and integers on the number line.

Grade Level Content Expectation:

- N.ME.05.10 Understand a fraction as a statement of division, e.g., $2 \div 3 = \frac{2}{3}$, using simple fractions and pictures to represent.

MS. 2. Recognize equivalent representations of a number, especially fractions, decimals and percents, and translate freely among representations.

Grade Level Content Expectation:

- N.ME.05.08 Understand the relative magnitude of ones, tenths, and hundredths and the relationship of each place value to the place to its right, e.g., 1 is 10 tenths, one tenth is 10 hundredths.
- N.ME.05.09 Understand percentages as parts out of 100, use % notation, and express a part of a whole as a percentage.
- N.ME.05.11 Given two fractions, express them as equivalent fractions with a common denominator, but not necessarily a least common denominator, e.g., $1/2=4/8$ and $3/4=6/8$ use denominators less than 12, or factors of 100.
- N.MR.05.22 Express fractions and decimals as percentages, and vice versa.
- N.ME.05.23 Express ratios in several ways, given applied situations, e.g., 3 cups to 5 people, 3:5, $3/5$; recognize and find equivalent ratios.

MS. 3. Distinguish between numbers that are used for counting, numbers that are used for ordering, numbers that are used for measuring and numbers that are used for naming.

MS. 4. Develop and refine strategies for estimating quantities, including fractional quantities, and evaluate the reasonableness and appropriateness of their estimates.

Grade Level Content Expectation:

- N.ME.05.08 Understand the relative magnitude of ones, tenths, and hundredths and the relationship of each place value to the place to its right, e.g., 1 is 10 tenths, one tenth is 10 hundredths.
- N.FL.05.20 Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness; use examples involving money.

MS. 5. Select appropriate representations for numbers, including integers and rational numbers, in order to simplify and solve problems.

Grade Level Content Expectation:

- N.FL.05.20 Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness; use examples involving money.

Michigan Content Standard:

3. Number Relationships

- Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Grade Level Benchmark:

MS. 1. Compare and order integers and rational numbers using relations of equality and inequality.

Grade Level Content Expectation:

- N.ME.05.08 Understand the relative magnitude of ones, tenths, and hundredths and the relationship of each place value to the place to its right, e.g., 1 is 10 tenths, one tenth is 10 hundredths.

MS. 2. Express a numerical comparison as ratios and rates.

Grade Level Content Expectation:

- N.ME.05.23 Express ratios in several ways, given applied situations, e.g., 3 cups to 5 people, 3:5, $\frac{3}{5}$ recognize and find equivalent ratios.

MS. 3. Distinguish between prime and composite numbers; identify factors, multiples, common factors and multiples, and relatively prime numbers; and apply divisibility tests to numbers.

Grade Level Content Expectation:

- N.MR.05.07 Find the prime factorization of numbers between 1 and 50, express in exponential notation, e.g., $24 = 2^3 \times 3^1$, and understand that every whole number can be expressed as a product of primes.

MS. 4. Explain the meaning of powers and roots of numbers and use calculators to compute powers and square roots.

Grade Level Content Expectation:

- N.MR.05.15 Multiply a whole number by powers of 10: 0.01, 0.1, 1, 10, 100, and 1000; identify patterns.

MS. 5. Apply their understanding of number relationships in solving problems.

Strand V: Numerical and Algebraic Operations and Analytical Thinking

Michigan Content Standard:

1. Operations and their Properties

- Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.

Grade Level Benchmark:

MS. 1. Use manipulatives and diagrams to model operations and their inverses with integers and rational numbers and relate the models to their symbolic expressions.

Grade Level Content Expectation:

- N.MR.05.01 Understand the meaning of division of whole numbers, with and without remainders; relate division to fractions and to repeated subtraction.
- N.MR.05.02 Relate division of whole numbers with remainders to the form $a = bq + r$, e.g., $34 \div 5 = 6 \text{ r } 4$, so $5 \cdot 6 + 4 = 34$; note remainder (4) is less than divisor (5).

MS. 2. Compute with integers, rational numbers and simple algebraic expressions using mental computation, estimation, calculators and paper-and-pencil; explain what they are doing and how they know which operations to perform in a given situation.

Grade Level Content Expectation:

- N.FL.05.04 Multiply a multi-digit number by a two-digit number; recognize and be able to explain common computational errors such as not accounting for place value.
- N.FL.05.06 Divide fluently up to a four-digit number by two-digit number. Multiply and divide fractions.
- N.FL.05.12 Find the product of two unit fractions with small denominators using area model.
- N.FL.05.13 Divide a fraction by a whole number and a whole number by a fraction, using simple unit fractions. Add and subtract fractions using common denominators.
- N.FL.05.14 Add and subtract fractions with unlike denominators of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 100, using the common denominator that is the product of the denominators of the 2 fractions, e.g.,

$$\begin{aligned} \frac{3}{8} + \frac{7}{10} &= \frac{(3 \cdot 10) + (7 \cdot 8)}{80} \\ &= \frac{30 + 56}{80} \\ &= \frac{86}{80} \end{aligned}$$

- N.FL.05.16 Divide numbers by 10's, 100's, 1000's, using mental strategies.

- N.MR.05.17 Multiply one-digit and two-digit whole numbers by decimals up to two decimal places. Solve applied problems with fractions.
- N.FL.05.20 Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness; use examples involving money.

MS. 4. Efficiently and accurately apply operations with integers, rational numbers, and simple algebraic expressions in solving problems.

Grade Level Content Expectation:

- N.MR.05.05 Solve applied problems involving multiplication and division of whole numbers. Solve applied problems with fractions.
- N.MR.05.19 Solve word problems that involve finding sums and differences of fractions with unlike denominators, using knowledge of equivalent fractions.
- N.FL.05.20 Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness; use examples involving money.

MS. 3. Describe the properties of operations with Rational Numbers and integers (e.g., closure; associative, commutative and distributive properties) and give examples of how they use those properties.

Michigan Content Standard:

2. Algebraic and Analytic Thinking

- Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Grade Level Benchmark:

MS. 1. Read and write algebraic expressions; develop original examples expressed verbally and algebraically; simplify expressions and translate between verbal and algebraic expressions; and solve linear equations and inequalities.

Grade Level Content Expectation:

- N.MR.05.03 Write mathematical statements involving division for given situations. Solve applied problems with fractions.
- N.FL.05.18 Given an applied situation involving addition and subtraction of fractions, write mathematical statements describing the situation.
- N.MR.05.21 Solve for the unknown in such equations as: $\frac{1}{4} + X = \frac{7}{12}$.

MS. 2. Represent algebraic concepts with geometric models (e.g., algebra tiles), physical models (e.g., balance beam), tables and graphs; and write algebraic expressions to correspond to the multiple representations.

MS. 3. Solve linear equalities and inequalities using algebraic and geometric methods, and use the context of the problem to interpret and explain their solutions.

MS. 4. Analyze problems modeled by linear functions, determine strategies for solving the problems and evaluate the adequacy of the solutions in the context of the problems.

MS. 5. Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.

Strand VI: Probability and Discrete Mathematics

Michigan Content Standard:

1. Probability

- Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations.

Grade Level Benchmark:

MS. 1. Describe events as likely or unlikely and give qualitative and quantitative descriptions of the degree of likelihood.

MS. 2. Describe probability as a measure of certainty ranging from 0 to 1 and conduct activities that allow them to express probabilities of simple events in mathematical terms.

MS. 3. Conduct experiments and give examples to illustrate the difference between dependent and independent events.

MS. 4. Explain the difference between probabilities determined from experiments or chance events (empirical) and probabilities derived mathematically (theoretical), and explain how the empirical probability changes for a large number of trials.

MS. 5. Conduct probability experiments and simulations to model and solve problems.

Michigan Content Standard:

2. Discrete Mathematics

- Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.

Grade Level Benchmark:

MS. 1. Use manipulatives, diagrams, and the fundamental theorem of counting to count permutations and combinations.

MS. 2. Use sets and set relationships to explore and solve simple algebraic and geometric problems.

MS. 3. Solve problems involving networks, for example planning delivery routes or counting paths between points.

MS. 4. Explore recurrence relations and iterations.

MS. 5. Continue to use manipulatives and drawings to model the concepts and procedures for the standard arithmetic algorithms, and develop and analyze their own and other students' algorithms to accomplish a task or solve a mathematical problem.

MS. 6. Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems) and decide upon a best solution (optimization problems).

Mathematics Revision 2006
Scope and Sequence by Grade
Grade Six

Strand I – Patterns, Relationships, Functions

Michigan Content Standard:

1. Patterns:

Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.

Grade Level Benchmark:

MS. 1. Describe, analyze and generalize patterns arising in a variety of contexts and express them in general terms.

MS. 2. Represent and record patterns in a variety of ways including tables, charts and graphs, and translate between various representations.

Grade Level Content Expectation:

- A.RP.06.08 Understand that graphs and tables can suggest relationships between quantities.

MS. 3. Use patterns and their generalizations to make and justify inferences and predictions.

MS. 4. Explore and describe visual and numeric patterns, including linear expressions, near-linear patterns and symmetric and spatial patterns.

MS. 5. Use patterns and generalizations to solve problems and explore new content.

Michigan Content Standard:

2. Variability and Change:

- Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

Grade Level Benchmark:

MS. 1. Identify and describe the nature of change; recognize change in more abstract and complex situations and explore different kinds of change, and patterns of variation.

MS. 2. Connect an initial state to a final state and generalize a rule that describes a pattern of change.

MS. 3. Begin to investigate applications in bivariate data and linear relationships and explore questions of what will happen to one quantity if another variable is changed.

MS. 4. Represent variability of change by ordered pairs, tables, graphs, and equations.

Grade Level Content Expectation:

- A.RP.06.10 Represent simple relationships between quantities, using verbal descriptions, formulas or equations, tables, and graphs, e.g. perimeter-side relationship for a square, distance-time graphs, and conversions such as feet to inches.

MS. 5. Differentiate between functions and relationships such as linear vs. not linear or continuous vs. non-continuous.

MS. 6. Continue to explore relationships arising from interesting contexts and use variables and relationships to solve mathematical problems.

Grade Level Content Expectation:

- A.PA.06.09 Graph and write equations for linear functions of the form $y = mx$ and solve related problems, e.g., given n chairs, the “leg function” is $f(n) = 4n$: if you have 5 chairs, how many legs? ; if you have 12 legs, how many chairs?

Strand II: Geometry and Measurement

Michigan Content Standard:

1. Shape and Shape Relationship

- Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

MS. 2. Generalize the characteristics of shapes and apply their generalizations to classes of shapes.

MS. 3. Derive generalizations about shapes and apply those generalizations to develop classifications of familiar shapes.

MS. 4. Construct familiar shapes using coordinates, appropriate tools (including technology), sketching and drawing two- and three-dimensional shapes.

Grade Level Content Expectation:

- M.PS.06.02 Draw patterns (of faces) for a cube and rectangular prism that, when cut, will cover the solid exactly (nets).
- G.SR.06.05 Use paper folding to perform basic geometric constructions of perpendicular lines, midpoints of line segments and angle bisectors, and justify informally.

MS. 5. Combine, dissect, and transform shapes.

Grade Level Content Expectation:

- G.TR.06.03 Understand the basic rigid motions the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems.
- G.TR.06.04 Understand and use simple compositions of basic rigid transformations, e.g., translation followed by a reflection.

MS. 6. Generalize about the common properties of similar, congruent, parallel and perpendicular shapes and verify their generalizations informally.

Grade Level Content Expectation:

- G.GS.06.01 Understand and apply basic properties of lines, angles, and triangles, including: triangle inequality relationships of vertical angles, complementary angles, supplementary angles, congruence of corresponding and alternate interior angles when parallel lines are cut by a transversal, and that such congruencies imply parallel lines locate interior and exterior angles of any triangle and use the property that a exterior angle of a triangle is equal to the sum of the remote (opposite) interior angles know that the sum of the exterior angles of a convex polygon is 360.

- G.GS.06.02 Understand that for polygons, congruence means corresponding sides and angles have equal measures.

Michigan Content Standard:

2. Position

- Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

Grade Level Benchmark:

MS. 3. Describe translations, reflections, rotations, and dilations using the language of transformations, and employ transformations to verify congruence of figures.

Grade Level Content Expectation:

- G.TR.06.03 Understand the basic rigid motions in the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems.
- G.TR.06.04 Understand and use simple compositions of basic rigid transformations, e.g., a translation followed by a reflection.

MS. 4. Locate the position of points or objects described by two or more conditions; locate all the points (locus) that satisfy a given condition.

Grade Level Content Expectation:

- A.RP.06.02 Plot ordered pairs of integers and use ordered pairs of integers to identify points in all four quadrants of the coordinate plane.

MS. 5. Use concepts of position, direction and orientation to describe the physical world and to solve problems.

Grade Level Content Expectation:

- G.TR.06.03 Understand the basic rigid motions in the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems.

Michigan Content Standard:

3. Measurement

- Students compare attributes of two objects, or of one with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

Grade Level Benchmark:

MS. 3. Estimate measures with a specified degree of accuracy and decide if an estimate or a measurement is “close enough”.

MS. 5. Use proportional reasoning and indirect measurements to draw inferences.

Grade Level Content Expectation:

- M.UN.06.01 Convert between basic units of measurement within a single measurement system, e.g., square inches to square feet.
- M.TE.06.03 Compute the volume and surface area of cubes and rectangular prisms given the lengths of their sides, using formulas.

MS. 6. Apply measurement to describe the real world and to solve problems.

Strand III: Data Analysis and Statistics

Michigan Content Standard:

1. Collection, Organization, Presentation of Data

- Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.

Grade Level Benchmark:

MS. 1 Collect and explore data through observation, measurement, surveys, sampling techniques, and simulations.

MS. 2 Organize data using tables, charts, graphs, spreadsheets and databases.

MS. 3 Present data using a variety of appropriate representations and explain why one representation is preferred over another or how a particular representation may bias the presentation.

Grade Level Content Expectation:

- A.RP.06.08 Understand that graphs and tables can suggest relationships between quantities.
- A.RP.06.10 Represent simple relationships between quantities, using verbal descriptions, formulas or equations, tables, and graphs, e.g. perimeter-side relationship for a square, distance-time graphs, and conversions such as feet to inches.

MS. 4 Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.

Michigan Content Standard:

2. Description and Interpretations

- Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.

Grade Level Benchmark:

MS. 1 Critically read data from tables, charts, or graphs and explain the source of the data and what the data represent.

MS. 2 Describe the shape of a data distribution and identify the center, the spread, correlations, and any outliers.

MS. 3 Draw, explain, and justify conclusions based on data.

MS. 4 Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the possible sources of bias in the data or their presentation.

MS. 5 Formulate questions and problems and gather and interpret data to answer those questions.

Michigan Content Standard:

3. Inference and Prediction

- Making and testing hypotheses is an essential technique for gaining new knowledge.

Grade Level Benchmark:

MS. 1 Make and test hypotheses.

MS. 2 Design experiments to model and solve problems using sampling, simulations and controlled investigations.

MS. 2 Design experiments to model and solve problems using sampling, simulations and controlled investigations.

MS. 3 Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.

MS. 4 Make predictions and decisions based on data, including interpolations and extrapolations.

MS. 5 Employ investigations, mathematical models and simulations to make inferences and predictions to answer questions and solve problems.

Strand IV: Number Sense and Numeration

Michigan Content Standard:

1. Concepts and Properties of Numbers

- Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.

Grade Level Benchmark:

MS. 1. Develop an understanding of integers and rational numbers and represent rational numbers in both fraction and decimal form.

Grade Level Content Expectation:

- N.ME.06.06 Represent rational numbers as fractions or terminating decimals when possible, and translate between these representations.
- N.ME.06.07 Understand that a fraction or a negative fraction is a quotient of two integers, e.g., $-8/3$ is -8 divided by 3 . Understand rational numbers and their location on the number line.

- N.ME.06.17 Locate negative rational numbers (including integers) on the number line; know that numbers and their negatives add to 0, and are on opposite's sides and at equal distance from 0 on a number line.
- N.ME.06.18 Understand that rational numbers are quotients of integers (nonzero denominators), e.g., a rational number is either a fraction or a negative fraction.

MS. 2. Extend their understanding of numeration systems to include decimal numeration, scientific numeration and non-decimal numeration systems.

Grade Level Content Expectation:

- N.ME.06.16 Understand and use integer exponents, excluding powers of negative numbers; express numbers in scientific notation.

MS. 3. Develop an understanding of the properties of the integer and rational number systems (e.g., order, density) and of the properties of special numbers including 0, 1 and, pi and the additive and multiplicative inverses.

Grade Level Content Expectation:

- N.ME.06.05 Order rational numbers and place them on the number line. Add and subtract integers and rational numbers.
- N.MR.06.08 Understand integer subtraction as the inverse of integer addition; add and subtract integers, using integers from 10 to -10.
- N.ME.06.17 Locate negative rational numbers (including integers) on the number line; know that numbers and their negatives add to 0, and are on opposite's sides and at equal distance from 0 on a number line.
- N.ME.06.19 Understand that 0 is an integer that is neither negative nor positive.
- N.ME.06.20 Know that the absolute value of a number is the value of the number, ignoring the sign; or is the distance of the number from 0.
- A.FO.06.13 Understand that multiplying or dividing both sides of an equation by the same non-zero number creates a new equation that has the same solutions.

MS. 4. Apply their understanding of number systems to model and solve mathematical and applied problems.

Michigan Content Standard:

2. Representation and Uses of Numbers

- Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Grade Level Benchmark:

MS. 1. Give geometric representations of fractions, prime and composite numbers, triangular and square numbers, and other number concepts; represent rational numbers and integers on the number line.

Grade Level Content Expectation:

- N.ME.06.05 Order rational numbers and place them on the number line.

MS. 2. Recognize equivalent representations of a number, especially fractions, decimals and percents, and translate freely among representations.

Grade Level Content Expectation:

- N.ME.06.06 Represent rational numbers as fractions or terminating decimals when possible, and translate between these representations.

MS. 3. Distinguish between numbers that are used for counting, numbers that are used for ordering, numbers that are used for measuring and numbers that are used for naming.

MS. 4. Develop and refine strategies for estimating quantities, including fractional quantities, and evaluate the reasonableness and appropriateness of their estimates.

Grade Level Content Expectation:

- N.FL.06.14 For applied situations, estimate the answers to calculations involving operations with rational numbers.

MS. 5. Select appropriate representations for numbers, including integers and rational numbers, in order to simplify and solve problems.

Grade Level Content Expectation:

- N.FL.06.13 Solve word problems involving percentages in such contexts as sales taxes and tips, and involving positive rational numbers.
- N.FL.06.14 For applied situations, estimate the answers to calculations involving operations with rational numbers.

Michigan Content Standard:

3. Number Relationships

- Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Grade Level Benchmark:

MS. 1. Compare and order integers and rational numbers using relations of equality and inequality.

Grade Level Content Expectation:

- N.ME.06.05 Order rational numbers and place them on the number line.
- N.ME.06.17 Locate negative rational numbers (including integers) on the number line; know that numbers and their negatives add to 0, and are on opposite's sides and at equal distance from 0 on a number line.

Grade Level Benchmark:

MS. 2. Express a numerical comparison as ratios and rates.

Grade Level Content Expectation:

- N.ME.06.11 Find equivalent ratios by scaling up or scaling down.
- A.PA.06.01 Solve applied problems involving rates, including speed, e.g., if a car is going 50 mph, how far will it go in 3 1/2 hours?

MS. 3. Distinguish between prime and composite numbers; identify factors, multiples, common factors and multiples, and relatively prime numbers; and apply divisibility tests to numbers.

MS. 4. Explain the meaning of powers and roots of numbers and use calculators to compute powers and square roots.

Grade Level Content Expectation:

- N.ME.06.16 Understand and use integer exponents, excluding powers of negative numbers; express numbers in scientific notation.

MS. 5. Apply their understanding of number relationships in solving problems.

Strand V: Numerical and Algebraic Operations and Analytical Thinking

Michigan Content Standard:

1. Operations and their Properties

- **Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.**

Grade Level Benchmark:

MS. 1. Use manipulatives and diagrams to model operations and their inverses with integers and rational numbers and relate the models to their symbolic expressions.

MS. 2. Compute with integers, rational numbers and simple algebraic expressions using mental computation, estimation, calculators and paper-and-pencil; explain what they are doing and how they know which operations to perform in a given situation.

Grade Level Content Expectation:

- N.FL.06.02 Given an applied situation involving dividing fractions, write a mathematical statement to represent the situation.
- N.FL.06.04 Multiply and divide any two fractions, including mixed numbers, fluently. Add and subtract integers and rational numbers.
- N.FL.06.09 Add, subtract, multiply, and divide integers between -10 and 10; use number line and strip models for addition and subtraction.
- N.FL.06.10 Add, subtract, multiply and divide positive rational numbers fluently. Solve decimal, percentage and rational number problems.
- N.FL.06.12 Calculate part of a number given the percentage and the number.
- N.FL.06.14 For applied situations, estimate the answers to calculations involving operations with rational numbers.

MS. 3. Describe the properties of operations with Rational Numbers and integers (e.g., closure; associative, commutative and distributive properties) and give examples of how they use those properties.

MS. 4. Efficiently and accurately apply operations with integers, rational numbers, and simple algebraic expressions in solving problems.

Grade Level Content Expectation:

- N.FL.06.13 Solve word problems involving percentages in such contexts as sales taxes and tips, and involving positive rational numbers.
- N.FL.06.15 Solve applied problems that use the four operations with appropriate decimal numbers.

- N.MR.06.01 Understand division of fractions as the inverse of multiplication, e.g., $4/5 \div 2/3 =$, then $2/3 * =4/5$, so $= 4/5 * 2/3 = 12/10$.
- N.FL.06.02 Given an applied situation involving dividing fractions, write a mathematical statement to represent the situation.
- N.MR.06.03 Solve for the unknown inequations such as:
 $1/4 \div = 1$, $3/4 \div = 1/4$ AND $1/2 - 1X$.
- A.FO.06.03 Use letters, with units, to represent quantities in a variety of contexts, e.g., lbs., k minutes, x cookies.
- A.FO.06.04 Distinguish between an algebraic expression and an equation.
- A.FO.06.05 Use standard conventions for writing algebraic expressions, e.g., $2x + 1$ means two times x, plus 1” and $2(x + 1)$ means “two times the quantity (x + 1)”.
- A.FO.06.06 Represent information given in words using algebraic expressions and equations.
- A.FO.06.07 Simplify expressions of the first degree by combining like terms, and evaluate using specific values. Represent linear functions using tables, equations, and graphs.
- A.RP.06.10 Represent simple relationships between quantities, using verbal descriptions, formulas or equations, tables, and graphs, e.g. perimeter-side relationship for a square, distance-time graphs, and conversions such as feet to inches Solve equations.
- A.FO.06.11 Relate simple linear equations with integer coefficients to particular contexts, e.g., $3x = 8$ or $x + 5 = 10$, and solve.

Michigan Content Standard:

2. Algebraic and Analytic Thinking

- Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Grade Level Benchmark:

MS. 1. Read and write algebraic expressions; develop original examples expressed verbally and algebraically; simplify expressions and translate between verbal and algebraic expressions; and solve linear equations and inequalities.

MS. 2. Represent algebraic concepts with geometric models (e.g., algebra tiles), physical models (e.g., balance beam), tables and graphs; and write algebraic expressions to correspond to the multiple representations.

MS. 3. Solve linear equalities and inequalities using algebraic and geometric methods, and use the context of the problem to interpret and explain their solutions.

Grade Level Content Expectation:

- A.FO.06.12 Understand that adding or subtracting the same number to both sides of an equation creates a new equation that has the same solution.
- A.FO.06.14 Solve equations of the form $ax + b = c$, e.g., $3x + 8 = 15$, by hand for positive integer coefficients less than 20, using calculators otherwise, and interpret the results.

MS. 4. Analyze problems modeled by linear functions, determine strategies for solving the problems and evaluate the adequacy of the solutions in the context of the problems.

Grade Level Content Expectation:

- A.PA.06.09 Graph and write equations for linear functions of the form $y = mx$ and solve related problems, e.g., given n chairs, the “leg function” is $f(n) = 4n$: if you have 5 chairs, how many legs? ; if you have 12 legs, how many chairs?

MS. 5. Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.

Grade Level Content Expectation:

- A.PA.06.01 Solve applied problems involving rates, including speed, e.g., if a car is going 50 mph, how far will it go in $3 \frac{1}{2}$ hours?

Strand VI: Probability and Discrete Mathematics

Michigan Content Standard:

1. Probability

- Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations.

Grade Level Benchmark:

MS. 1. Describe events as likely or unlikely and give qualitative and quantitative descriptions of the degree of likelihood.

Grade Level Content Expectation:

- D.PR.06.01 Express probabilities as fractions, decimals or percentages between 0 and 1; know that 0 probability means an event will not occur, and that probability 1 means an event will occur.
- D.PR.06.02 Compute probabilities of events from simple experiments with equally likely outcomes, e.g., tossing dice, flipping coins, spinning spinners, by listing all possibilities and finding the fraction that meets given conditions.

MS. 2. Describe probability as a measure of certainty ranging from 0 to 1 and conduct activities that allow them to express probabilities of simple events in mathematical terms.

Grade Level Content Expectation:

- D.PR.06.01 Express probabilities as fractions, decimals or percentages between 0 and 1; know that 0 probability means an event will not occur, and that probability 1 means an event will occur.
- D.PR.06.02 Compute probabilities of events from simple experiments with equally likely outcomes, e.g., tossing dice, flipping coins, spinning spinners, by listing all possibilities and finding the fraction that meets given conditions.

MS. 3. Conduct experiments and give examples to illustrate the difference between dependent and independent events.

Grade Level Content Expectation:

- D.PR.06.02 Compute probabilities of events from simple experiments with equally likely outcomes, e.g., tossing dice, flipping coins, spinning spinners, by listing all possibilities and finding the fraction that meets given conditions.

MS. 5. Conduct probability experiments and simulations to model and solve problems.

Grade Level Content Expectation:

- D.PR.06.02 Compute probabilities of events from simple experiments with equally likely outcomes, e.g., tossing dice, flipping coins, spinning spinners, by listing all possibilities and finding the fraction that meets given conditions.

Michigan Content Standard:

2. Discrete Mathematics

- Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.

Grade Level Benchmark:

MS. 2. Use sets and set relationships to explore and solve simple algebraic and geometric problems.

MS. 5. Continue to use manipulatives and drawings to model the concepts and procedures for the standard arithmetic algorithms, and develop and analyze their own and other students' algorithms to accomplish a task or solve a mathematical problem.

Mathematics Revision 2006
Scope and Sequence by Grade
Grade Seven

Strand I – Patterns, Relationships, Functions

Michigan Content Standard:

1. Patterns:

- Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.

Grade Level Benchmark:

MS. 1. Describe, analyze and generalize patterns arising in a variety of contexts and express them in general terms.

MS. 2. Represent and record patterns in a variety of ways including tables, charts, and graphs, and translate between various representations.

Grade Level Content Expectation:

- A.RP.07.02 Represent directly proportional and linear relationships using verbal descriptions, tables, graphs and formulas, and translate among these representations.

MS. 4. Explore and describe visual and numeric patterns, including linear expressions, near-linear patterns and symmetric and spatial patterns.

Grade Level Content Expectation:

- A.PA.07.01 Recognize when information given in a table, graph or formula suggests a proportional or linear relationship.
- A.RP.07.02 Represent directly proportional and linear relationships using verbal descriptions, tables, graphs and formulas, and translate among these representations.

MS. 5. Use patterns and generalizations to solve problems and explore new content.

Michigan Content Standard:

2. Variability and Change:

- Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

Grade Level Benchmark:

MS. 1. Identify and describe the nature of change; recognize change in more abstract and complex situations and explore different kinds of change, and patterns of variation.

Grade Level Content Expectation:

- A.PA.07.03 Given a directly proportional or linear situation, graph and interpret the slope and intercept(s) in terms of the original situation; evaluate $y = kx$ for specific x values, given k , e.g., weight vs. volume of water, base cost plus cost per unit.
- N.FL.07.03 Calculate rates of change, including speed.
- N.FL.07.05 Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation $a/b = c/d$; know how to see patterns about proportional situations in tables.

MS. 2. Connect an initial state to a final state and generalize a rule that describes a pattern of change.

MS. 3. Begin to investigate applications in bivariate data and linear relationships, and explore questions of what will happen to one quantity if another variable is changed.

Grade Level Content Expectation:

- N.FL.07.03 Calculate rates of change, including speed.
- N.FL.07.05 Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation $a/b = c/d$; know how to see patterns about proportional situations in tables.

MS. 4. Represent variability of change by ordered pairs, tables, graphs, and equations.

Grade Level Content Expectation:

- A.RP.07.02 Represent directly proportional and linear relationships using verbal descriptions, tables, graphs and formulas, and translate among these representations.

MS. 5. Differentiate between functions and relationships such as linear vs. not linear or continuous vs. non-continuous.

Grade Level Content Expectation:

- A.PA.07.09 Recognize inversely proportional relationships in contextual situations; know that quantities are inversely proportional if their product is constant, e.g., the length and width of a rectangle with fixed area, and that an inversely proportional relationship is of the form $y = k/x$ where k is some non-zero number.
- A.RP.07.10 Know that the graph of $y = k/x$ is not a line; know its shape, and know that it crosses neither the x nor the y -axis.

MS. 6. Continue to explore relationships arising from interesting contexts, and use variables and relationships to solve mathematical problems.

Strand II: Geometry and Measurement

Michigan Content Standard:

1. Shape and Shape Relationship

- Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

Grade Level Benchmark:

MS. 4. Construct familiar shapes using coordinates, appropriate tools (including technology), sketching and drawing two- and three-dimensional shapes.

Grade Level Content Expectation:

- G.SR.07.01 Use a ruler and other tools to draw squares, rectangles, triangles and parallelograms with specified dimensions.

MS. 7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.

Grade Level Content Expectation:

- A.PA.07.04 For directly proportional or linear situations, solve applied problems using graphs and equations; e.g., the heights and volume of a container with uniform cross-section; height of water in a tank being filled at a constant rate; degrees Celsius and degrees Fahrenheit; distance and time under constant speed.
- A.PA.07.05 Understand and use directly proportional relationships of the form $y = mx$, and distinguish from linear relationships of the form $y = mx + b$, b non-zero; understand that in a directly proportional relationship between two quantities one quantity is a constant multiple of the other quantity.

Michigan Content Standard:

2. Position

- Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

Grade Level Benchmark:

MS. 1. Locate and describe objects in terms of their position, including compass directions, Cartesian coordinates, latitude and longitude, and midpoints.

MS. 2. Locate and describe objects in terms of their orientation and relative position, including coincident, collinear, parallel, perpendicular; differentiate between fixed (e.g./ N-S- E-W) and relative (e.g., right-left) orientations; recognize and describe examples of bilateral and rotational symmetry.

MS. 5. Combine, dissect, and transform shapes.

Grade Level Content Expectation:

- G.SR.07.02 Use compass and straightedge to perform basic geometric constructions: the perpendicular bisector of a segment, an equilateral triangle, and the bisector of an angle; understand informal justifications.

MS. 7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems

Grade Level Content Expectation:

- A.PA.07.04 For directly proportional or linear situations, solve applied problems using graphs and equations; e.g., the heights and volume of a container with uniform cross-section; height of water in a tank being filled at a constant rate; degrees Celsius and degrees Fahrenheit; distance and time under constant speed.
- A.PA.07.05 Understand and use directly proportional relationships of the form $y = mx$, and distinguish from linear relationships of the form $y = mx + b$, b non-zero; understand that in a directly proportional relationship between two quantities one quantity is a constant multiple of the other quantity.

Michigan Content Standard:

3. Measurement

- Students compare attributes of two objects, or of one with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

Grade Level Benchmark:

MS. 1. Select and use appropriate tools; measure objects using standard units in both the metric and common systems, and measure angles in degrees.

Grade Level Content Expectation:

- G.SR.07.01 Use a ruler and other tools to draw squares, rectangles, triangles and parallelograms with specified dimensions.
- G.SR.07.02 Use compass and straightedge to perform basic geometric constructions: the perpendicular bisector of a segment, an equilateral triangle, and the bisector of an angle; understand informal justifications.

MS. 4. Interpret measurements and recognize that two objects may have the same measurement on one attribute (e.g., area), but not necessarily on another (e.g., perimeter).

Grade Level Content Expectation:

- G.TR.07.06 Understand and use the fact that when two triangles are similar with scale factor of r , their areas are related by a factor of r^2 .

MS. 5. Use proportional reasoning and indirect measurements to draw inferences.

Grade Level Content Expectation:

- G.TR.07.03 Understand that in similar polygons, corresponding angles are congruent and the ratios of corresponding sides are equal; understand the concepts of similar figures and scale factor.
- G.TR.07.04 Solve problems about similar figures and scale drawings.
- G.TR.07.05 Show that two triangles are similar using the criteria: corresponding angles are congruent (AAA similarity); the ratios of two pairs of corresponding sides are equal and the included angles are congruent (SAS similarity); ratios of all pairs of corresponding sides are equal (SSS similarity); use these criteria to solve problems and to justify arguments.
- G.TR.07.06 Understand and use the fact that when two triangles are similar with scale factor of r , their areas are related by a factor of r^2 .
- N.FL.07.05 Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation $a/b = c/d$; know how to see patterns about proportional situations in tables.

MS. 6. Apply measurement to describe the real world and to solve problems.

Strand III: Data Analysis and Statistics

Michigan Content Standard:

1. Collection, Organization, Presentation of Data

- Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.

Grade Level Benchmark:

MS. 1 Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.

MS. 2 Organize data using tables, charts, graphs, spreadsheets, and databases.

Grade Level Content Expectation:

- D.RE.07.01 Represent and interpret data using circle graphs, stem and leaf plots, histograms, and box-and-whisker plots, and select appropriate representation address specific questions.
- D.AN.07.02 Create and interpret scatter plots and an estimated line of best fit to answer questions about the data.

MS. 3 Present data using a variety of appropriate representations and explain why one representation is preferred over another or how a particular representation may bias the presentation.

Grade Level Content Expectation:

- D.RE.07.01 Represent and interpret data using circle graphs, stem and leaf plots, histograms, and box-and whisker plots, and select appropriate representation to address specific questions.
- D.AN.07.02 Create and interpret scatter plots and use an estimated line of best fit to answer questions about the data.

MS. 4. Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.

Michigan Content Standard:

2. Description and Interpretations

- Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.

Grade Level Benchmark:

MS. 1 Critically read data from tables, charts or graphs and explain the source of the data and what the data represent.

MS. 1 Describe the shape of a data distribution and identify the center, the spread, correlations, and any outliers.

Grade Level Content Expectation:

- D.AN.07.04 Find and interpret the median, quartiles, and interquartile range of a given set of data.

MS. 3 Draw, explain and justify conclusions based on data.

Grade Level Content Expectation:

- D.RE.07.01 Represent and interpret data using circle graphs, stem and leaf plots, histograms, and box-and-whisker plots, and select appropriate representation to address specific questions.
- D.AN.07.02 Create and interpret scatter plots and use an estimated line of best fit to answer questions about the data.

MS. 4 Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the possible sources of bias in the data or their presentation.

Grade Level Content Expectation:

- D.RE.07.01 Represent and interpret data using circle graphs, stem and leaf plots, histograms, and box-and-whisker plots, and select appropriate representation to address specific questions.
- D.AN.07.02 Create and interpret scatter plots and use an estimated line of best fit to answer questions about the data.

MS. 5 Formulate questions and problems and gather and interpret data to answer those questions.

Michigan Content Standard:

3. Inference and Prediction

- Making and testing hypotheses is an essential technique for gaining new knowledge.

Grade Level Benchmark:

MS. 1 Make and test hypotheses.

MS. 3 Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.

MS. 4 Make predictions and decisions based on data, including interpolations and extrapolations.

MS. 5 Employ investigations, mathematical models and simulations to make inferences and predictions to answer questions and solve problems.

Strand IV: Number Sense and Numeration

Michigan Content Standard:

1. Concepts and Properties of Numbers

- Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.

Grade Level Benchmark:

MS. 1. Develop an understanding of integers and rational numbers and represent rational numbers in both fraction and decimal form.

MS. 2. Extend their understanding of numeration systems to include decimal numeration, scientific numeration and non-decimal numeration systems.

Grade Level Content Expectation:

- N.MR.07.06 Understand the concept of square root and cube root, and estimate using calculators.

MS. 3. Develop an understanding of the properties of the integer and rational number systems (e.g., order, density) and of the properties of special numbers including 0, 1 and, and the additive and multiplicative inverses.

Grade Level Content Expectation:

- A.PA.07.11 Understand and use basic properties of real numbers: additive and multiplicative identities, additive and multiplicative inverses, commutativity, associativity, and the distributive property of multiplication over addition. Combine algebraic expressions and solve equations.
- A.FO.07.12 Add, subtract and multiply simple algebraic expressions of the first degree, e.g., $(92x + 8y) - 5x + y$, or $-2x(5x - 4)$, and justify using properties of real numbers.

MS. 4. Apply their understanding of number systems to model and solve mathematical and applied problems.

Michigan Content Standard:

2. Representation and Uses of Numbers

- Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Grade Level Benchmark:

MS. 1. Give geometric representations of fractions, prime and composite numbers, triangular and square numbers, and other number concepts; represent rational numbers and integers on the number line.

MS. 2. Recognize equivalent representations of a number, especially fractions, decimals and percents, and translate freely among representations.

MS. 3. Distinguish between numbers that are used for counting, numbers that are used for ordering, numbers that are used for measuring and numbers that are used for naming.

MS. 4. Develop and refine strategies for estimating quantities, including fractional quantities, and evaluate the reasonableness and appropriateness of their estimates.

Grade Level Content Expectation:

- N.MR.07.06 Understand the concept of square root and cube root, and estimate using calculators. Compute with rational numbers.
- N.FL.07.09 Estimate results of computations with rational numbers.

MS. 5. Select appropriate representations for numbers, including integers and rational numbers, in order to simplify and solve problems.

Grade Level Content Expectation:

- N.FL.07.02 Solve problems involving derived quantities.
- N.FL.07.05 Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation $a/b = c/d$; know how to see patterns about proportional situations in tables.

Michigan Content Standard:

3. Number Relationships

- Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Grade Level Benchmark:

MS. 1. Compare and order integers and rational numbers using relations of equality and inequality.

MS. 2. Express a numerical comparison as ratios and rates.

Grade Level Content Expectation:

- N.ME.07.01 Understand derived quantities such as density, velocity, and weighted averages. Understand and solve problems involving rates, ratios, and proportions.

- N.MR.07.04 Convert ratio quantities between different systems of units, such as feet per second to miles per hour. Understand and apply directly proportional relationships; relate to linear relationships.
- A.PA.07.01 Recognize when information in a table, graph or formula suggests a proportional or linear relationship.
- A.RP.07.02 Represent directly proportional and linear relationships using verbal descriptions, tables, graphs and formulas, and translate among these representations.
- A.PA.07.03 Given a directly proportional or linear situation, graph and interpret the slope and intercept(s) in terms of the original situation; evaluate $y = kx$ for specific x values, given k , e.g., weight vs. volume of water, base cost plus cost per unit.
- A.PA.07.05 Understand and use directly proportional relationships of the form $y = mx$, and distinguish from linear relationships of the form $y = mx + b$, b non-zero; understand that in a directly proportional relationship between two quantities one quantity is a constant multiple of the other quantity.
- A.PA.07.06 Calculate the slope from the graph of a linear function as the ratio of “rise/run” for a pair of points on the graph, and express the answer as a fraction and a decimal; understand that linear functions have slope that is a constant rate of change.
- A.PA.07.07 Represent linear functions in the form $y = x + b$, $y = mx$, and $y = mx + b$, and graph, interpreting slope and y -intercept.

MS. 3. Distinguish between prime and composite numbers; identify factors, multiples, common factors and multiples, and relatively prime numbers; and apply divisibility tests to numbers.

MS. 4. Explain the meaning of powers and roots of numbers and use calculators to compute powers and square roots.

Grade Level Content Expectation:

- N.MR.07.06 Understand the concept of square root and cube root, and estimate using calculators.

MS. 5. Apply their understanding of number relationships in solving problems.

Strand V: Numerical and Algebraic Operations and Analytical Thinking

Michigan Content Standard:

1. Operations and their Properties

- Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.

Grade Level Benchmark:

MS. 1. Use manipulatives and diagrams to model operations and their inverses with integers and rational numbers and relate the models to their symbolic expressions.

MS. 2. Compute with integers, rational numbers and simple algebraic expressions using mental computation, estimation, calculators and paper-and-pencil; explain what they are doing and how they know which operations to perform in a given situation.

Grade Level Content Expectation:

- N.FL.07.08 Add, subtract, multiply and divide negative rational numbers.
- N.FL.07.09 Estimate results of computations with rational numbers.

MS. 3. Describe the properties of operations with Rational Numbers and integers (e.g., closure; associative, commutative and distributive properties) and give examples of how they use those properties.

Grade Level Content Expectation:

- A.PA.07.11 Understand and use basic properties of real numbers: additive and multiplicative identities, additive and multiplicative inverses, commutativity, associativity, and the distributive property of multiplication over addition.

MS. 4. Efficiently and accurately apply operations with integers, rational numbers, and simple algebraic expressions in solving problems.

Grade Level Content Expectation:

- N.FL.07.02 Solve problems involving derived quantities.
- N.FL.07.05 Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation $a/b = c/d$; know how to see patterns about proportional situations in tables.
- N.FL.07.07 Solve problems involving operations with integers.
- N.FL.07.09 Estimate results of computations with rational numbers.

Michigan Content Standard:

2. Algebraic and Analytic Thinking

- Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Grade Level Benchmark:

MS. 1. Read and write algebraic expressions; develop original examples expressed verbally and algebraically; simplify expressions and translate between verbal and algebraic expressions; and solve linear equations and inequalities.

Grade Level Content Expectation:

- A.FO.07.12 Add, subtract and multiply simple algebraic expressions of the first degree, e.g., $(92x + 8y) - 5x + y$, or $-2x(5x - 4)$, and justify using properties of real numbers.

MS. 2. Represent algebraic concepts with geometric models (e.g., algebra tiles), physical models (e.g., balance beam), tables and graphs; and write algebraic expressions to correspond to the multiple representations.

MS. 3. Solve linear equalities and inequalities using algebraic and geometric methods, and use the context of the problem to interpret and explain their solutions.

Grade Level Content Expectation:

- A.FO.07.08 Know that the solution to a linear equation corresponds to the point at which its graph crosses the x-axis. Combine algebraic expressions and solve equations.
- A.FO.07.13 From applied situations, generate and solve linear equations of the form $ax + b = c$ and $ax + b = cx + d$, and interpret solutions.

MS. 4. Analyze problems modeled by linear functions, determine strategies for solving the problems and evaluate the adequacy of the solutions in the context of the problems.

Grade Level Content Expectation:

- A.PA.07.04 For directly proportional or linear situations, solve applied problems using graphs and equations; e.g., the heights and volume of a container with uniform cross-section; height of water in a tank being filled at a constant rate; degrees Celsius and degrees Fahrenheit; distance and time under constant speed.
- A.PA.07.07 Represent linear functions in the form $y = x + b$, $y = mx$, and $y = mx + b$, and graph, interpreting slope and yintercept.

MS. 5. Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.

Grade Level Content Expectation:

- A.PA.07.04 For directly proportional orlinear situations, solve applied problems using graphs and equations; e.g., the heights and volume of a container with uniform cross-section; height of water in a tank being filled at a constant rate; degrees Celsius and degrees Fahrenheit; distance and time under constant speed.

Strand VI: Probability and Discrete Mathematics

Michigan Content Standard:

1. Probability

- Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations.

Grade Level Benchmark:

MS. 3. Conduct experiments and give examples to illustrate the difference between dependent and independent events.

MS. 4. Explain the difference between probabilities determined from experiments or chance events (empirical) and probabilities derived mathematically (theoretical), and explain how the empirical probability changes for a large number of trials.

Michigan Content Standard:

2. Discrete Mathematics

- Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.

Grade Level Benchmark:

MS. 1. Use manipulative, diagrams, and the fundamental theorem of counting to count permutations and combinations.

MS. 2. Use sets and set relationships to explore and solve simple algebraic and geometric problems.

MS.3. Solve problems involving networks, for example planning delivery routes or counting paths between points.

MS.4. Explore recurrence relations and iterations.

MS. 5. Continue to use manipulative and drawings to model the concepts and procedures for the standard arithmetic algorithms, and develop and analyze their own and other students' algorithms to accomplish a task or solve a mathematical problem.

MS. 6. Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems) and decide upon a best solution (optimization problems).

Mathematics Revision 2006
Scope and Sequence by Grade
Grade Eight

Strand I – Patterns, Relationships, Functions

Michigan Content Standard:

1. Patterns:

- Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.

Grade Level Benchmark:

HS. 1. Analyze and generalize mathematical patterns including sequences, series, and recursive patterns.

HS. 2. Analyze, interpret and translate among representations of patterns including tables, charts, graphs, matrices and vectors.

Grade Level Content Expectation:

- A.RP.08.01 Identify and represent linear functions, quadratic functions, and other simple functions including inverse functions.
- $(y = k/x)$, cubics $(y = ax^3)$, roots $(y = x)$, and exponentials $(y = a^x, a > 0)$, using tables, graphs, and equations.

HS. 3. Study and employ mathematical models of patterns to make inferences, predictions and decisions.

HS. 4. Explore patterns (graphic, numeric, etc.) characteristic of families of functions; explore structural patterns within systems of objects, operations or relations.

Grade Level Content Expectation:

- A.RP.08.01 Identify and represent linear functions, quadratic functions, and other simple functions including inverse functions $(y = k/x)$, cubics $(y = ax^3)$, roots $(y = x)$, and exponentials $(y = a^x, a > 0)$, using tables, graphs, and equations.
- A.PA.08.02 For basic functions, e.g. simple quadratics, direct and indirect variation, and population growth, describe how changes in one variable affect the others.

HS. 5. Use patterns and reasoning to solve problems and explore new content.

Michigan Content Standard:

2. Variability and Change:

- Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

Grade Level Benchmark:

HS. 1. Identify and describe the nature of change and begin to use the more formal language such as rate of change, continuity, limit, distribution and deviation.

HS. 2. Develop a mathematical concept of function and recognize that functions display characteristic patterns of change (e.g., linear, quadratic, exponential).

Grade Level Content Expectation:

- A.R.P.08.04 Use the vertical line test to determine if a graph represents a function in one variable.

HS. 3. Expand their understanding of function to include non-linear functions, composition of functions, inverses of functions, and piecewise- and recursively- defined functions.

Grade Level Content Expectation:

- A.PA.08.02 For basic functions, e.g., simple quadratics, direct and indirect variation, and population growth, describe how changes in one variable affect the others.

HS. 4. Represent functions using symbolism such as matrices, vectors and functional representation ($f(x)$).

HS. 5. Differentiate and analyze classes of functions including linear, power, quadratic, exponential, circular, and trigonometric functions, and realize that many different situations can be modeled by a particular type of function.

Grade Level Content Expectation:

- A.RP.08.01 Identify and represent linear functions, quadratic functions, and other simple functions including inverse functions ($y = k/x$), cubics ($y = ax^3$), roots ($y = x$), and exponentials ($y = a^x$, $a > 0$) using tables, graphs, and equations.
- A.PA.08.03 Recognize basic functions in problem context, e.g. area of a circle is πr^2 , volume of a sphere is $\frac{4}{3}\pi r^3$, and represent them using tables, graphs, and formulas.
- A.R.P..08.05 Relate quadratic functions in factored form and vertex form to their graphs and vice versa; in particular, note that solutions of a quadratic equation are the x-intercepts of the corresponding quadratic function.
- A.R.P. 08.06 Graph factorable quadratic functions, finding where the graph intersects the x axis and the coordinates of the vertex; use words "parabola" and "roots"; include

functions in vertex form and those with leading coefficient -1, e.g., $y = x^2 - 36$, $y = (x - 2)^2 - 9$; $y = -x^2$; $y = -(x - 3)^2$.

HS. 6. Increase their use of functions and mathematical models to solve problems in context.

Strand II: Geometry and Measurement

Michigan Content Standard:

1. Shape and Shape Relationship

- Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

Grade Level Benchmark:

HS. 2. Determine necessary and sufficient conditions for the existence of a particular shape and apply those conditions to analyze shapes.

Grade Level Content Expectation:

- G.SR.08.03 Understand the definition of a circle; know and use the formulas for circumference and area of a circle to solve problems.
- G.SR.08.05 Solve applied problems involving areas of triangles, quadrilaterals, and circles.
- G.SR.08.07 Understand the concept of surface area and find the surface area of prisms, cones, spheres, pyramids, and cylinders.

HS. 4. Draw and construct shapes in two and three dimensions and analyze and justify the steps of their constructions.

Grade Level Content Expectation:

- G.SR.08.08 Sketch a variety of two-dimensional representations of three-dimensional solids, including: orthogonal views (top, front, and side), picture views projective or isometric) and nets, and use such two-dimensional representations to help solve problems.

HS. 5. Study transformations of shapes using isometrics, size transformations, and coordinate mappings.

Grade Level Content Expectation:

- G.TR.08.09 Understand the definition of a dilation from a point in the plane and relate it to the definition of similar polygons.
- G.TR.08.10 Understand and use reflective and rotational symmetries of two-dimensional shapes and relate them to transformations to solve problems.

HS. 6. Compare and analyze shapes and formally establish the relationships among them, including congruence, similarity, parallelism, perpendicularity, and incidence.

HS. 7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.

Grade Level Content Expectation:

- G.GS.08.01 Understand at least one proof of the Pythagorean Theorem; use the Pythagorean Theorem and its converse to solve applied problems, including perimeter, area, and volume problems.
- G.SR.08.05 Solve applied problems involving areas of triangles, quadrilaterals, and circles.
- G.SR.08.08 Sketch a variety of two-dimensional representations of three dimensional solids, including: orthogonal views (top, front, and side), picture views projective or isometric) and nets, and use such two-dimensional representations to help solve problems.

Michigan Content Standard:

2. Position

- Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

HS. 5. Use concepts of position, direction, and orientation to describe the physical world and to solve problems.

Grade Level Content Expectation:

- G.L.O. 08.02 Find the distance between two points on the coordinate plane using the distance formula; recognize that the distance formula is an application of the Pythagorean Theorem.

Michigan Content Standard:

3. Measurement

- Students compare attributes of two objects, or of one with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

Grade Level Benchmark:

HS. 1. Select and use appropriate tools; make accurate measurements using both metric and common units, and measure angles in degrees and radians.

HS. 2. Continue to make and apply measurements of length, mass, weight, time, temperature, area, volume, angle; classify objects according to their dimensions.

Grade Level Content Expectation:

- G.SR.08.03 Understand the definition of a circle; know and use the formulas for circumference and area of a circle to solve problems.

- G.SR.08.04 Find area and perimeter of complex figures by subdividing them into basic shapes (quadrilaterals, triangles, circles).
- G.SR.08.05 Solve applied problems involving areas of triangles, quadrilaterals, and circles.
- G.SR.08.06 Know the volume formulas for generalized cylinders ((area of base) x height), generalized cones and pyramids ($\frac{1}{3}$ (area of base) x height) and spheres $\frac{4}{3}\pi$ radius³, and apply them to solve problems.
- G.SR.08.07 Understand the concept of surface area and find the surface area of prisms, cones, spheres, pyramids, and cylinders.

Grade Level Content Expectation:

- G.SR.08.04 Find area and perimeter of complex figures by subdividing them into basic shapes (quadrilaterals, triangles, circles).

HS. 5. Use proportional reasoning and indirect measurements, including applications of trigonometric ratios to measure inaccessible distances and to determine derived measures such as density.

HS. 6. Apply measurement to describe the real world and to solve problems.

Strand III: Data Analysis and Statistics

Michigan Content Standard:

1. Collection, Organization, Presentation of Data

- Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.

Grade Level Benchmark:

HS. 1. Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.

HS. 2. Organize data using tables, charts, graphs, spreadsheets, and data bases.

HS. 3. Present data using the most appropriate representation and give a rationale for their choice; show how certain representations may skew the data or bias the presentation.

HS. 4. Identify what data are needed to answer a particular question or solve a given problem and design and implement strategies to obtain, organize, and present those data.

Michigan Content Standard:

2. Description and Interpretations

- Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.

Grade Level Benchmark:

HS. 1. Critically read data from tables, charts, or graphs and explain the source of the data and what the data represent.

HS. 2. Describe the shape of a data distribution and determine measures of central tendency, variability and correlation.

Grade Level Content Expectation:

- D.AN.08.01 Determine which measure of central tendency (mean, median, mode) best represents a data set, e.g. salaries, home prices for answering certain questions, and justify the choice made.

HS. 3. Use data and their characteristics to draw and support conclusions.

HS. 4. Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the sources of bias and measures taken to eliminate such bias.

Grade Level Content Expectation:

- D.AN.08.02 Recognize practices of collecting and displaying data that may bias the presentation or analysis.

HS. 5. Formulate questions and problems and gather and interpret data to answer those questions.

Strand IV: Number Sense and Numeration

Michigan Content Standard:

1. Concepts and Properties of Numbers

- Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.

Grade Level Benchmark:

HS. 1. Develop an understanding of irrational, real, and complex numbers.

Grade Level Content Expectation:

- N.ME.08.03 Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals, e.g. $0.1 = 1/9$; $0.3 = 1/3$.
- N.ME.08.01 Understand the meaning of a square root of a number and its connection to the square whose area is the number; understand the meaning of a cube root and its connection to the volume of a cube.
- N.ME.08.02 Understand meanings for zero and negative integer exponents.

HS. 4. Apply their understanding of number systems to model, and solve mathematical and applied problems.

Michigan Content Standard:

2. Representation and Uses of Numbers

- Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Grade Level Benchmark:

HS. 1. Give decimal representations of rational and irrational numbers and coordinate and vector representations of complex numbers.

Grade Level Content Expectation:

- N.ME.08.03 Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals e.g. $0.1 = 1/9$; $0.3 = 1/3$.

HS. 4. Apply estimation to increasingly complex situations.

Grade Level Content Expectation:

- N.ME.08.04 Understand that irrational numbers are those that cannot be expressed as the quotient of two integers, and cannot be represented by terminating or repeating decimals; approximate the position of familiar irrational numbers, e.g. 2, 3 on the number line.
- N.FL.08.05 Estimate and solve problems with square roots and cube roots, using calculators.
- N.FL.08.06. Find square roots of perfect squares and approximate the square roots of non-perfect squares by locating between consecutive integers, e.g. 130 is between 11 and 12.

Michigan Content Standard:

3. Number Relationships

- Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Grade Level Benchmark:

HS. 1. Compare and order real numbers and compare rational approximations to exact values.

Grade Level Content Expectation:

- N.ME.08.03 Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals e.g., $0.1 = 1/9$; $0.3 = 1/3$.
- N.ME.08.04 Understand that irrational numbers are those that cannot be expressed as the quotient of two integers, and cannot be represented by terminating or repeating decimals; approximate the position of familiar irrational numbers, e.g., 2, 3 on the number line.

HS. 2. Express numerical comparisons as ratios and rates.

Grade Level Content Expectation:

- N.MR.08.07 Understand percent increase and percent decrease in both sum and product form, e.g, 3% increase of a quantity x is $x + .03x = 1.03x$.
- A.PA.08.02 For basic functions, e.g, simple quadratics, direct and indirect variation, and population growth, describe how changes in one variable affect the others.

HS. 3. Extend the relationships of primes, factors, multiples and divisibility in an algebraic setting.

Grade Level Content Expectation:

- N.ME.08.01 Understand the meaning of a square root of a number and its connection to the square whose area is the number; understand the meaning of a cube root and its connection to the volume of a cube.

- N.ME.08.02 Understand meanings for zero and negative integer exponents.
- A.FO. 08.07 Recognize and apply the common formulas:
 $(a + b)^2 = a^2 + 2ab + b^2$
 $(a - b)^2 = a^2 - 2ab + b^2$
 $(a + b)(a - b) = a^2 - b^2$; represent geometrically.
- A.FO.08.08 Factor simple quadratic expressions with integer coefficients, e.g., $x^2 + 6x + 9$, $x^2 + 2x - 3$ and $x^2 - 4$; solve simple quadratic equations e.g., $x^2 = 16$ or $x^2 = 5$ (by taking square roots); $x^2 - 2x = 15$ (by factoring) verify solutions by evaluation.

HS. 4. Express number relationships using positive and negative rational exponents, logarithms and radicals.

Grade Level Content Expectation:

- A.RP.08.01 Identify and represent linear functions, quadratic functions, and other simple functions including inverse functions ($y = k/x$), cubics ($y = ax^3$), roots ($y = x$), and exponentials ($y = a^x$, $a > 0$), using tables, graphs, and equations.

HS. 5. Apply their understanding of number relationships in solving problems.

Grade Level Content Expectation:

- N.FL.08.09 Solve problems involving compounded interest or multiple discounts.
- N.MR.08.10 Calculate weighted averages such as course grades, consumer price indices, and sports ratings.
- N.MR.08.11 Solve problems involving ratio units, such as miles per hour, dollars per pound, or persons per square mile.
- A.FO. 08.09 Solve applied problems involving simple quadratic equations.

Strand V: Numerical and Algebraic Operations and Analytical Thinking

Michigan Content Standard:

1. Operations and their Properties

- Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.

Grade Level Benchmark:

HS. 1. Present and explain geometric and symbolic models for operations with real and complex numbers and algebraic expressions.

HS. 2. Compute with real numbers, complex numbers, algebraic expressions, matrices and vectors using technology, and for simple instances, with paper-and-pencil algorithms.

HS. 3. Describe the properties of operations with numbers, algebraic expressions, vectors and matrices, and make generalizations about the properties of given mathematical systems.

HS. 4. Efficiently and accurately apply operations with real numbers, complex numbers, algebraic expressions, matrices, and vectors in solving problems.

Michigan Content Standard:

2. Algebraic and Analytic Thinking

- Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Grade Level Benchmark:

HS. 1. Identify important variables in a context, symbolize them and express their relationships algebraically.

Grade Level Content Expectation:

- A.FO.08.10 Understand that to solve the equation $f(x) = g(x)$ means to find all values of x for which the equation is true; e.g., determine whether a given value, or values from a given set, is a solution of an equation (0 is a solution of $3x^2 + 2 = 4x + 2$, but 1 is not a solution).
- A.FO.08.12 Solve linear inequalities in one and two variables, and graph the solution sets.

HS. 2. Identify important variables in a context, symbolize them, and express their relationships algebraically.

HS. 3. Solve linear equations and inequalities algebraically and non-linear equations using graphing, symbol-manipulating or spreadsheet technology; and solve linear and non-linear systems using appropriate methods.

Grade Level Content Expectation:

- A.FO.08.10 Understand that to solve the equation $f(x) = g(x)$ means to find all values of x for which the equation is true; e.g., determine whether a given value, or values from a given set, is a solution of an equation (0 is a solution of $3x^2 + 2 = 4x + 2$, but 1 is not a solution).
- A.FO.08.11 Solve simultaneous linear equations in two variables, by graphing, by substitution and by linear combination; estimate solutions using graphs; include examples with no solutions and infinitely many solutions.
- A.FO.08.12 Solve linear inequalities in one and two variables, and graph the solution sets. Understand solutions and solve equations, simultaneous equations, and linear inequalities.
- A.FO.08.13 Set up and solve applied problems involving simultaneous linear equations and linear inequalities.

HS. 4. Analyze problems that can be modeled by functions, determine strategies for solving the problems and evaluate the adequacy of the solutions in the context of the problems.

HS. 5. Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.

Strand VI: Probability and Discrete Mathematics

Michigan Content Standard:

1. Probability

- Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgements about claims that are made in probabilistic situations.

Grade Level Benchmark:

HS. 1. Develop an understanding of randomness and chance variation and describe chance and certainty in the language of probability.

HS. 2. Give a mathematical definition of probability and determine the probabilities of more complex events, and generate and interpret probability distributions.

Grade Level Content Expectation:

- **D.PR. 08.05 Understand the relationship of probability to relative frequency.**

HS. 3. Analyze events to determine their dependence or independence and calculate probabilities of compound events.

Grade Level Content Expectation:

- D.PR.08.04 Apply the Basic Counting Principle to find total number of outcomes possible for independent and dependent events, and calculate the probabilities using organized lists or tree diagrams.
- D.PR.08.06 Understand the difference between independent and dependent events, and recognize common misconceptions involving probability, e.g, Alice rolls a 6 on a die three times in a row; she is just as likely to roll a 6 on the 4th roll as she was on any previous roll.

HS. 4. Use sampling and simulations to determine empirical probabilities and, when appropriate, compare them to the corresponding theoretical probabilities; understand and apply the law of large numbers.

Grade Level Content Expectation:

- D.PR.08.06 Understand the difference between independent and dependent events, and recognize common misconceptions involving probability, e.g., Alice rolls a 6 on a die three times in a row; she is just as likely to roll a 6 on the 4th roll as she was on any previous roll.

HS. 5. Conduct probability experiments and simulations, to model and solve problems, including compound events.

Grade Level Content Expectation:

- D.PR.08.03 Compute relative frequencies from a table of experimental results for a repeated event and be able to answer questions about the results, using relationship of probability to relative frequency.
- D.PR. 08.05 Understand the relationship of probability to relative frequency.
- D.AN.08.07 Compute relative frequencies from a table of experimental results for a repeated event; understand the relationship of experimental probability to relative frequency; answer questions regarding the results.

Michigan Content Standard:

2. Discrete Mathematics

- Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.

Grade Level Benchmark:

HS. 1. Derive and use formulas for calculating (the number of) permutations and combinations.

Grade Level Content Expectation:

- D.PR.08.04 Apply the Basic Counting Principle to find total number of outcomes possible for independent and dependent events, and calculate the probabilities using organized lists or tree diagrams.

HS. 2. Use sets and set relationships to represent algebraic and geometric concepts.